

Appendix A.

Here, we develop analytical results for the accuracy rates and RT distributions for the model. This section could be safely skipped by readers who are less interested in the mathematical details. The mathematical notation we use is summarized in Table A1. Note that the *conditional* probabilities in the table are conditional on the event that the search have not terminated prior to the *k*'th selection.

Table A1: mathematical notations

Parameter	Definition
n	Display set size
m	The probability of a motor error
$T_{min}^{yes}, T_{min}^{no}, \gamma$	The shift for "target present" and "target absent" responses and the (common) rate of the shifted exponential residual time
w_{target}	The salience weight of the target
Δw_{quit}	The increment to the weight of the quit unit following each rejection of distracter
θ, ν, σ	The threshold, drift rate and standard deviation of noise in the identification process
p_k^{quit}	the conditional probability that the quit unit is selected on a target present trial on the <i>k</i> 'th iteration: , <i>k</i> =1,2,... <i>n</i>
p_k^{target}	the conditional probability that the target is selected on a target present trial on the <i>k</i> 'th selection: , <i>k</i> =1,2,... <i>n</i>
P_k^{quit}	the probability that the quit unit is selected on a target present trial on the <i>k</i> 'th iteration: , <i>k</i> =1,2,... <i>n</i>
P_k^{target}	the probability that the target is selected on a target present trial on the <i>k</i> 'th selection: , <i>k</i> =1,2,... <i>n</i>
$P_k^{survive}$	The probability that a target present trial didn't terminate after the <i>k</i> 'th item selection.
$q_k^{quit}, Q_k^{quit}, Q_k^{survive}$	Same a 'P's' but for target absent trials, <i>k</i> =1,2,... <i>n</i> +1
$f_k^{yes}(t), f_k^{no}(t)$	The density of the RT for trials that terminated with a yes or no response after <i>k</i> item identifications, <i>k</i> =1,2,... <i>n</i>
$(f_W, F_W)(t \nu, \sigma, \theta)$	The probability density/cumulative density function of a Wald distribution with drift ν , standard deviation of noise, σ and threshold θ
$(f_{EW}, F_{EW})(t \nu, \sigma, \theta, \gamma)$	The probability density/cumulative density function of an ex-Wald distribution which is the sum of a Wald distribution with parameters ν, σ, θ and an exponential distribution with rate γ
$(f_{sEW}, F_{sEW})(t \nu, \sigma, \theta, \gamma, s)$	The probability density/cumulative density function of a shifted ex-Wald distribution with parameters $\nu, \sigma, \theta, \gamma$ and shift <i>s</i>
P_{hit}	The probability of a hit
P_{miss}	The probability of a miss
P_{CR}	The probability of a correct rejection
P_{FA}	The probability of a false alarm
$(f, F)_{hit}(t)$	The probability density/cumulative density functions for hits
$(f, F)_{miss}(t)$	The probability density/cumulative density functions for misses
$(f, F)_{CR}(t)$	The probability density/cumulative density functions for correct rejections
$(f, F)_{FA}(t)$	The probability density/cumulative density functions for false alarms
$\Phi(t)$	The cumulative density function of a standard normal distribution

Note, that when it is necessary to distinguish explicitly between different set sizes we

add to the above notation a superscript denoting the set size. For example P_{CR}^n denotes to correct rejection rate for set size n .

Consider first target present trials: If the search has not terminated until the $k - 1$ 'th selection then the weight of the quit unit prior to the k 'th selection is $(k - 1)\Delta w_{quit}$. Furthermore, as $k-1$ distracters have already been inhibited, $n - k$ activated distracters and the target show up on the saliency map. Therefore, the total weight of the saliency map is: $n - k + w_{target}$. It follows that, the conditional probability of search termination (i.e. selection of the quit unit) is:

$$p_k^{quit} = \frac{(k - 1)\Delta w_{quit}}{n - k + w_{target} + (k - 1)\Delta w_{quit}} \tag{Eq. 1}$$

And the conditional probability to select the target is:

$$\begin{aligned} p_k^{target} &= (1 - p_k^{quit}) * \frac{w_{target}}{n - k + w_{target}} \\ &= \frac{w_{target}}{n - k + w_{target} + (k - 1)\Delta w_{quit}} \end{aligned} \tag{Eq. 2}$$

The (non-conditional) probabilities for selecting the quit unit and the target on the k 'th selection respectively are obtained by multiplying the conditional probabilities with the survival term:

$$P_k^{quit} = p_k^{quit} * P_{k-1}^{survive} ; P_k^{target} = p_k^{target} * P_{k-1}^{survive} \tag{Eq. 3}$$

Finally, the search will not terminate following the k 'th selection (assuming it hasn't terminated until the $k - 1$ selection) if and only if neither a termination decision is made, nor the target is selected on the k 'th selection:

$$\begin{aligned} P_k^{survive} &= P_{k-1}^{survive} - P_k^{quit} - P_k^{target} \\ &= P_{k-1}^{survive} (1 - p_k^{quit} - p_k^{target}) \\ &= P_{k-1}^{survive} \frac{n - k}{n - k + w_{target} + (k - 1)\Delta w_{quit}} \end{aligned} \tag{Eq. 4}$$

Beginning with $P_0^{survive} = 1$ (the search couldn't have terminated after 0 item selections) we find by iteratively applying Eq. 13 that:

$$P_k^{survive} = \prod_{l=1}^k \frac{n - l}{n - l + w_{target} + (l - 1)\Delta w_{quit}} \tag{Eq. 5}$$

From Eq. 10,11,12,14 it follows that:

$$P_k^{quit} = \prod_{l=1}^{k-1} \frac{n-l}{n-l+w_{target}+(l-1)\Delta w_{quit}} \cdot \frac{(k-1)\Delta w_{quit}}{n-k+w_{target}+(k-1)\Delta w_{quit}} \tag{Eq. 6}$$

$$P_k^{target} = \prod_{l=1}^{k-1} \frac{n-l}{n-l+w_{target}+(l-1)\Delta w_{quit}} \cdot \frac{w_{target}}{n-k+w_{target}+(k-1)\Delta w_{quit}} \tag{Eq. 7}$$

We also denote $P_{n+1}^{quit} = 0$ since for target present displays there are at most n selections.

Consider next, target absent trials: In this case events are simpler as the only possibility to quit a search is to select the quit unit. If the search didn't terminate until the $k - 1$ 'th search then the weight of the saliency map just before the k 'th selection is $n - k + 1$. Thus:

$$q_k^{quit} = \frac{(k-1)\Delta w_{quit}}{n-k+1+(k-1)\Delta w_{quit}} \tag{Eq. 8}$$

$$Q_k^{quit} = q_k^{quit} * Q_{k-1}^{survive} \tag{Eq. 9}$$

The search will 'survive' the k 'th selection if the quit unit is not selected hence,

$$\begin{aligned} Q_k^{survive} &= Q_{k-1}^{survive} - Q_k^{quit} = Q_{k-1}^{survive}(1 - q_k^{quit}) \\ &= Q_{k-1}^{survive} \frac{n-k+1}{n-k+1+(k-1)\Delta w_{quit}} \end{aligned} \tag{Eq. 10}$$

Proceeding as before we thus obtain:

$$Q_k^{survive} = \prod_{l=1}^k \frac{n-l+1}{n-l+1+(l-1)\Delta w_{quit}} \tag{Eq. 11}$$

$$q_k^{quit} = \prod_{l=1}^{k-1} \frac{n-l+1}{n-l+1+(l-1)\Delta w_{quit}} \cdot \frac{(k-1)\Delta w_{quit}}{n-k+1+(k-1)\Delta w_{quit}} \tag{Eq. 12}$$

Next we consider RT's. The duration of a single identification is distributed $Wald(\nu, \sigma, \theta)$. The density and the cumulative density functions are given by:

$$f_W(t|\nu, \sigma, \theta) = \begin{cases} \sqrt{\frac{(\theta/\sigma)^2}{2\pi t^3}} * e^{-\frac{(\theta/\sigma)^2(t-\theta/\nu)^2}{2(\theta/\nu)^2 t}}, & t \geq 0 \\ 0, & t < 0 \end{cases} \tag{Eq. 13}$$

$$F_W(t|\nu, \sigma, \theta) = \begin{cases} \Phi\left(\sqrt{\frac{(\theta/\sigma)^2}{t}}\left(\frac{t}{\theta/\nu} - 1\right)\right) + e^{-\frac{2(\theta/\sigma)^2}{\theta/\nu}} \Phi\left(-\sqrt{\frac{(\theta/\sigma)^2}{t}}\left(\frac{t}{\theta/\nu} + 1\right)\right), & x > 0 \\ 0, & x \leq 0 \end{cases} \quad \text{Eq. 14}$$

The duration for identifying k items is the sum of k independent single-item identification duration samples and is thus distributed $Wald(\nu, \sigma, k\theta)$. Recall that the residual time is distributed as a shifted exponential with shift T_{min} and rate γ and is independent of the identification times. Thus, when the trial is terminated after k identifications the RT density is the convolution of a Wald and a shifted exponential density which is a shifted ex-Wald distribution. Schwartz (2002) derived analytic formulas for the density and the cumulative distributions of the ex-Wald distribution, which we readily utilize by incorporating a shift.

Following Schwartz (2002), two cases should be distinguished: In the first, $\nu^2 \geq 2\gamma\sigma^2$. In this case we define $d = \sqrt{\nu^2 - 2\gamma\sigma^2}$ and the density of the ex-Wald distribution is:

$$f_{EW}(t|\nu, \sigma, \theta, \gamma) = \gamma e^{-\gamma t + \frac{\theta(\nu-d)}{\sigma^2}} F_W(t|d, \sigma, \theta) \quad \text{Eq. 15}$$

In the second case, $\nu^2 < 2\gamma\sigma^2$ and the derivation requires complex function theory. We define $d' = \sqrt{2\gamma\sigma^2 - \nu^2}$ and then:

$$f_{EW}(t|\nu, \sigma, \theta, \gamma) = \begin{cases} \gamma e^{-\frac{(\theta-\nu t)^2}{2\sigma^2 t}} \text{Re}\left[w\left(\frac{d'\sqrt{t}}{\sqrt{2}\sigma} + i\frac{\theta}{\sqrt{2}t\sigma}\right)\right], & t \geq 0 \\ 0, & t < 0 \end{cases} \quad \text{Eq. 16}$$

In this case the real part of the complex function $w(z)$ is calculated. Denote by $\text{erf}(z)$, the complex error function. Then:

$$w(z) = e^{-z^2} [1 - \text{erf}(-iz)] \quad \text{Eq. 17}$$

For both cases the cumulative density is given by:

$$F_{EW}(t|\nu, \sigma, \theta, \gamma) = F_W(t|\nu, \sigma, \theta) - \frac{1}{\gamma} f_{EW}(t|\nu, \sigma, \theta, \gamma) \quad \text{Eq. 18}$$

For shifted ex-Wald distribution, a shift should be incorporated so:

$$(f, F)_{SEW}(t|\nu, \sigma, \theta, \gamma, s) = (f, F)_{EW}(t - s|\nu, \sigma, \theta, \gamma) \quad \text{Eq. 19}$$

The distribution of RT's conditional on k item selections is given by:

$$(f, F)_k^{yes/no}(t) = (f, F)_{SEW}(t|\nu, \sigma, k\theta, \gamma, T_{min}^{yes/no}) \quad \text{Eq. 20}$$

After all those preparations we can finally derive the equations for the behavioral Appendix A

measures of accuracy rates and RT densities. A hit is obtained if either: (i) the target is identified and a motor error is avoided, or (ii) the quit unit is selected, but a motor error occurs. Thus:

$$P_{hit} = (1 - m) * \sum_{k=1}^n P_k^{target} + m * \sum_{k=1}^n P_k^{quit} \tag{Eq. 21}$$

Similarly, a miss occurs when (i) the target is identified but a motor error occurs or (ii) when the quit unit is selected (prior to target identification) and a motor error is avoided. Thus:

$$P_{miss} = m * \sum_{k=1}^n P_k^{target} + (1 - m) * \sum_{k=1}^n P_k^{quit} \tag{Eq. 22}$$

$$= 1 - P_{hit}$$

Substituting Eq. 15,16 into Eq. 30, 31 give closed forms for the probabilities of hits and misses.

The RT density function for hits/misses is obtained as a mixture of the densities $f_k(t)$ with mixture weight that are the conditional (on a hit/miss respectively) probabilities that the search terminated after exactly k identifications. A hit occurs after k identification if either the target is the k 'th identification and a motor error is avoided, or if it is decided to terminate the search after the k 'th selection and a motor error occurs. Hence

$$(f, F)_{hit}(t) = \frac{\sum_{k=1}^n ((1 - m)P_k^{target} + mP_{k+1}^{quit})(f, F)_k^{yes}(t)}{P_{hit}} \tag{Eq. 23}$$

Similarly, a miss occurs after k identifications if the target is the k 'th selection but a motor error occurs or the quit unit is the $k + 1$ 'th selection and a motor error is avoided. Thus:

$$(f, F)_{miss}(t) = \frac{\sum_{k=1}^n (mP_k^{target} + (1 - m)P_{k+1}^{quit})(f, F)_k^{no}(t)}{P_{miss}} \tag{Eq. 24}$$

For target absent displays the situation is similar. A correct rejection occurs if and only if a motor error is avoided and a false alarm occurs if and only if a motor error occurs. Thus:

$$P_{CR} = 1 - m \tag{Eq. 25}$$

$$P_{FA} = m \tag{Eq. 26}$$

$$\begin{aligned}
 (f, F)_{CR}(t) &= \frac{\sum_{k=1}^n (1 - m) Q_{k+1}^{quit} (f, F)_k(t)}{P_{CR}} \\
 &= \sum_{k=1}^n Q_{k+1}^{quit} (f, F)_k^{yes}(t)
 \end{aligned}
 \tag{Eq. 27}$$

$$\begin{aligned}
 (f, F)_{FA}(t) &= \frac{\sum_{k=1}^n m Q_{k+1}^{quit} (f, F)_k(t)}{P_{FA}} \\
 &= \sum_{k=1}^n Q_{k+1}^{quit} (f, F)_k^{no}(t)
 \end{aligned}
 \tag{Eq. 28}$$

Note that the RT distributions for the ‘non residual processing time’ for correct rejections and false alarm are identical in the current model as the sole determinant of the outcome is the event of a motor error which is independent of the display processing time. The total RT distributions for these trials differ only due to the shift in the residual RT: T_{min}^{yes} for the former and T_{min}^{no} for the later.

As an illustration Figure 3 demonstrates how the RT probability density functions for hits and for correct rejections (henceforth, CR) are generated as mixture distributions. The Figure corresponds to the average subject of the spatial configuration task and to set size $n = 6$. The Figure shows 6 individual ‘hit’ and 6 individual ‘CR’ defective RT-distributions that correspond to the event that the hit or CR trial terminates after k display items are identified (where $k = 1, 2, \dots, 6$). A defective distribution is a multiplication of a distribution by a positive constant, so that the area below the distribution is not 1, but rather that constant. In the Figure, each of the mixture-component defective distributions is a shifted ex-Wald distribution (with parameters $\nu, \sigma, k * \theta, \gamma, T_{min}^{yes/no}$) that is multiplied by the conditional probability that the trial (yielding a hit or a CR) terminated after exactly k identifications, the coefficient of $(f, F)_k^{yes/no, 6}(t)$ in Eq. 32, 26 respectively. The thick probability density functions (real, not defective) are obtained as a mixture of the individual distributions. These distributions are just the functional, that is the ‘point-wise vertical sum’, of the individual components. Although difficult to read from the figure, it is important to note that, the 6 component defective densities are identical up to a shift, for hits and for correct rejections, but they are weighted differently. As set size increases, for example to $n = 12$, the same components (with updated weights), will be adjoined with new components for $6 \leq k \leq 12$, to generated mixture probabilities.