No News is News: Nonignorable Nonresponse in Roll-Call Data Analysis

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Online appendix

1 Abstention, absences, and missing values in world legislatures

In this section, we provide an overview of the incidence of non-response in legislatures around the world. We base our conclusions on a convenience sample collected from legislatures that make their roll-calls public online. In Table A, we distinguish among three types of missing values: explicit abstentions in which legislators actively choose not to vote either Aye or Nay; official absences in which legislators are physically absent from the floor; and a residual category that captures, among others, instances in which legislators were present but did not vote. The rates of *abstention*, *absence*, and *present but did not vote* correspond to the percentage of empty legislator-by-vote cells.

Table A: Abstention, absences, and missing values in legislatures around the world

		Average rates		
Country	Period	Abstention	Official absence	Other
Argentina	1984–1997	1.63	0	42.83[a]
Argentina	2005–2007		0	30.00[b]
Australia	1996–1998	0	0	17.61[a]
Belgium	1995–1999	6.00	10.00	
Brazil	1990–1995	31.00	1.00	0
Brazil	1995–1998	21.18	1.18	0
Brazil	2002–2007	0.60	29.00	9.00
Canada	1994–1997	0	37.73	0
Canada	1997–2000		39.00	
Canada[c]	2000–2004		21.00	
Chile	1997–1998	1.32		45.98[a]
Chile	1998–2000	1.48		39.39[a]
Czech Rep	1993–1996	10.58	29.42	5.68
Czech Rep	1996–1998	1.50	13.00	12.00
Czech Rep	1998–2000	1.57	14.41	12.23
Ecuador	1998–2002	11.59	19.64	4.39
Guatemala	1994–1995	0	16.87	4.82
Guatemala	1996–1999	0	16.48	10.82

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Guatemala	2000	0	6.28	2.34
Ireland	2002–2007			31.00[a]
Ireland	1989–1992			33.00[a]
Ireland	1992–1997			33.00[a]
Ireland	1997–2002			26.00[a]
Mexico	1998–2000	1.95	10.55	0
New Zealand	1990–1993	0	0	31.40[a]
New Zealand	1993–1994	0	0	21.12[a]
Nicaragua	1/00–9/00	0	0.32	45.65
Norway	2001–2005			47.00[a]
Norway	1993–1997			36.00[a]
Norway	1997–2001			43.00[a]
Peru	3/99–6/00	2.71	24.66	2.68
Peru	8/00–12/00	4.80	21.59	2.80
Peru	8/01–10/01	1.67	21.12	3.32
Philippines (House)	1995–1997	0.07	38.45	0
Philippines (Sen.)	1995	0.36	29.66	0
Poland	1997–1999	2.27	11.02	1.42
Switzerland	1993–2003	2.00	22.00	0
EU Parliament	1979–1984	2.23	29.40	27.89
EU Parliament	1984–1989	1.61	17.45	35.92
EU Parliament	1989–1994	1.07	32.81	29.71
EU Parliament	1994–1999	1.47	31.76	18.68
EU Parliament	1999–2004	3.00	22.12	4.80
UK	1992–1997			35.00[a]
UK	1997–2001			40.00[a]
UK	2001–2005			34.00[a]
Israel	1996–1999	0.50	78.00	0.10
Israel	1992–1996	0.60	76.00	0.30
Israel	1999–2003	1.00	70.00	0.20
Israel	10/99–11/99	1.06	58.93	0.20
Israel	1999[d]	0.52	74.68	0.08
Israel	2003–2006	0.78	73.49	0.09
Israel	2006–2008	0.70	70.18	0.10
US 64th (House)[e]	1915–1917		20.24	0.82
US 73rd (House)	1933–1934		9.62	2.93
US 93rd (House)	1973–1974		11.98	0.13
US 103rd (House)	1993–1995		4.00	0.02
US 106th (House)	1999–2000		5.20	0.09
US 109th (House)	2005–2006		4.00	0.06
US 64th (Sen.)	1915–1917		33.25	0.02
US 73rd (Sen.)	1933–1934		15.30	0
US 93rd (Sen.)	1973–1974		12.80	0.09
US 106th (Sen.)	1999–2000		2.42	0
US 109th (Sen.)	2005–2006		2.88	0

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[a]May include absences; [b]may include absences and abstentions; [c]legislators that served part-time in the session were dropped; [d]budget votes; [e]paired votes of absent legislators counted as Ayes/Nays

Table B: Subsample of 15 votes from a realization of different non-response processes

		Parameters	Observed choices														
A	No NAs	No missingness	1	0	1	1	1	1	1	0	0	0	1	1	1	1	0
B	MCAR	$\boldsymbol{\phi} = k$	1	0	1	·	·	1	·	·	0	0	1	1	·	1	·
C	MAR + D	$\boldsymbol{\phi} \perp \boldsymbol{\theta}$	·	0	1	1	1	·	1	·	0	0	·	1	1	·	0
D	MAR	$\boldsymbol{\phi} = f(\boldsymbol{\theta})$	1	·	1	·	1	1	1	·	0	0	1	1	1	1	·
E	MNAR I	$\boldsymbol{\phi} = f(\mathbf{Z})$	1	0	1	1	·	·	1	0	0	0	·	1	1	·	0
F	MNAR II	$\boldsymbol{\phi} = f(\mathbf{Z}, \boldsymbol{\theta})$	1	·	1	1	1	1	1	0	0	0	1	1	1	1	·

(1=Aye, 0=Nay, ·=Non-response)

2 Illustration of non-ignorable non-response mechanisms

In this section, we use a stylized example to illustrate the consequences of basing inferences about parameters on roll-calls with missing values under different non-response mechanisms. Consider the records of two legislators with ideal points at $x_1 = -0.5$ and $x_2 = 0.5$ who are asked to vote on 500 bills (Aye=1, Nay=0.) We assume that the process that drives vote choice is such that the probability of voting Aye increases on ideology; specifically, $\Pr(Z = 1|x) = \Phi_\theta(x)$, where $\theta = x$.

The top row in Table B shows a subsample (15 randomly chosen bills) of a realization of this process for the rightist legislator with ideal point $x_2 = 0.5$. From realized 1's and 0's ($\mathbf{z}$) we can estimate the ideal points of these two individuals through a transformation of their average voting record, i.e, $\widehat{x}_i = \Phi^{-1}(\bar{x}_i)$. For any one particular realization $\mathbf{z}$ of $\mathbf{Z}$, estimated values $\widehat{\mathbf{x}}$ will differ from the true values $\mathbf{x}$ because of randomness in the vote choice process, but Rubin's conditions entail that $\widehat{\mathbf{x}}$ will be an unbiased estimator of $\mathbf{x}$ if all data are observed or if non-response is ignorable.

Consider now five different probabilistic non-response processes (rows B through F in Table B). The first process is "missing completely at random" with a probability of non-response by any legislator on any one vote fixed at a constant value (i.e., $\phi = k$, where $k \in (0, 1)$ is constant). Row B shows the pattern of *observed* data $\mathbf{y}_1$ generated by this non-response process. The second process (row C) is "missing at random" with parameter distinctness, which in this case means that the probability of non-response is constant across votes cast *by the same legislator* but not across legislators, and that the legislator-specific probability of non-response is independent of the legislators' ideal points ($\boldsymbol{\phi} \perp \boldsymbol{\theta}$). The third process (D) is "missing at random" *without* parameter distinctness; thus, the probability of non-response varies across legislators (though still not across votes within legislator) and is proportional to the legislators' ideal points ($\boldsymbol{\phi} = f(\boldsymbol{\theta})$). Note that parameter distinctness does not hold in this case because the ideal points $\mathbf{x}$ inform *both* the vote choice process *and* the non-response process. The fourth process (E) is "not missing at random", since there is a strictly positive probability of non-response if vote choice is Aye, but the probability of non-response is 0 if vote choice is Nay ($\boldsymbol{\phi} = f(\mathbf{Z})$). The final process (F) is also "not missing at random", but here the probability of non-response is positive if vote choice is Aye for the leftist legislator, positive if vote choice is Nay for the rightist legislator, and 0 otherwise ($\boldsymbol{\phi} = f(\mathbf{Z}, \boldsymbol{\theta})$).

Table C inspects the consequences of ignoring the process of non-response, which entails drawing inferences about $\mathbf{x}$ exclusively from the *observed* data. The numerical cells in Table C report the sampling distribution of $\widehat{\mathbf{x}}$ and root MSE statistics over 10,000 draws based on the theoretical processes of *vote choice* and *non-response* described in the previous paragraphs. The second column summarizes the known characteristics of the non-response process and the third column displays

Table C: Sampling distribution of $\widehat{\mathbf{x}}$ (mean, standard error, and root mean square error) based on observed data under different non-response processes

			$x_1 = -0.5$			$x_2 = 0.5$		
Process	Conditions	Ignorable?	Mean	SE	RMSE	Mean	SE	RMSE
A	No NAs	Yes	−0.501	0.059	0.055	0.501	0.058	0.055
B	MCAR	Yes	−0.502	0.068	0.071	0.501	0.067	0.071
C	MAR + D	Yes	−0.501	0.071	0.071	0.501	0.071	0.071
D	MAR	No	−0.502	0.107	0.105	0.502	0.070	0.071
E	MNAR I	No	−0.909	0.071	0.415	0.072	0.069	0.434
F	MNAR II	No	−0.910	0.071	0.416	0.909	0.071	0.415

whether Rubin's non-ignorability conditions hold.

In row A, where all data happen to be observed, estimates of $\mathbf{x}$ that ignore the non-response process are unbiased and maximally efficient. Modeling the non-response-generating process in this trivial case would yield no payoff, as we would be conditioning parameter estimates on an indicator matrix $\mathbf{M}$ that does not vary ($\mathbf{M}$ holds 0s only, since all votes are observed). For the two other ignorable mechanisms (rows B and C), the effect of ignoring non-response is mild efficiency loss in estimating $\mathbf{x}$, as can be seen from increases in the standard errors and root MSE statistics of $\widehat{\mathbf{x}}$.

Instead, ignoring a non-ignorable non-response process is consequential. For the MAR process without parameter distinctness (row D), we see that the estimate $\widehat{x}_1$ is still unbiased, but with a noticeably larger standard error for $\widehat{x}_1$. For this particular example with two legislators, modeling non-response under the assumption of non-distinctness would mostly yield improvements in efficiency. Incidentally, the rate of random non-response of the rightist legislator is smaller than the rate of non-response of the leftist legislator, which explains why $\widehat{x}_2$ is a more efficient estimate of x_2 than $\widehat{x}_1$ is of x_1.

Finally, we see from the two MNAR processes in rows E and F that the main effect of assuming ignorability is very pronounced bias in $\widehat{\mathbf{x}}$. When Aye votes are more likely to be missing (row E), both ideal points are underestimated by about half the distance between x_1 and x_2; ignoring non-response in this case misses the "true" ideal points of legislators 1 and 2 but yields a correct inference about the degree of polarization between these two. When Aye votes are more likely to be missing for the leftist legislator, and Nay votes are more likely to be missing for the rightist legislator (row F), the leftist's ideal point is severely underestimated and the rightist's ideal point is severely overestimated. As a consequence, ignoring non-response yields an inflated sense of polarization between the positions of these two legislators. More problematically, the standard errors of $\widehat{\mathbf{x}}$ remain relatively narrow, underscoring the fact that by ignoring non-response in a "not missing at random" process we may recover a very wrong estimate with rather high certainty. This point is confirmed by the vast increase in the root MSE statistics of these two processes.

3 Indifference

In this section, we consider an abstention-generating mechanism driven by *indifference*, as recounted by Poole and Rosenthal (1997, ch. 10). In Poole and Rosenthal's account, legislators may choose non-response if they consider the utility differential between voting Aye and voting Nay—i.e., y^*_{ij} in the IRT model—to be trivially small. This event can happen under two conditions, *either* when the distance between status quo and bill proposal is minimal *or* when the distance between status quo and bill proposal is meaningful but the legislator's own ideological position is more or less equidistant from these two points. In both of these circumstances, indifferent legislators would be more likely to abstain.

To model this mechanism, we assume that a legislator abstains when the utility differential on any given vote drops below a certain individual-specific tolerance threshold γ_i, a parameter measured on the utility scale that we estimate from data. We further assume that tolerance thresholds $\boldsymbol{\gamma}$ and ideal points $\mathbf{x}$ are uncorrelated, which implies that the abstention process is MAR. To see this, consider that legislators abstain because *without regard for how they would have voted had they considered their options carefully* (a) they do not consider it worth their time to vote on a bill that might change the status quo ever so slightly or (b) they perceive the Aye and Nay positions as consequential, but equally distant. Adding these assumptions to the spatial logic underlying IRT allows us to construct a model of voting behavior under abstentions generated by indifference. Thus, we stipulate that legislator i registers a vote on bill j ($m_{ij} = 1$) only if the absolute value of his utility differential is larger than his tolerance threshold, i.e., only if $|y^*_{ij}| > \gamma_i$; otherwise $m_{ij} = 0$. This assumption allows us to derive the conditions that lead to observation of positive votes ($y_{ij} = 1$), negative votes ($y_{ij} = 0$), and missing values ($y_{ij} = \text{NA}$) in the roll-call matrix:

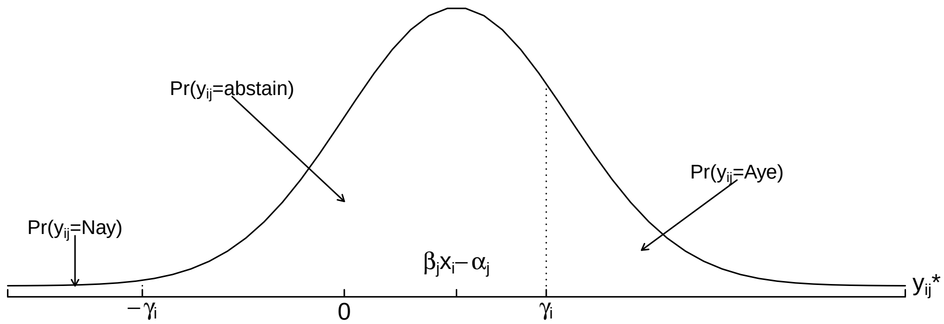
$$y_{ij} = \begin{cases} 1 & \text{if} \quad \gamma_i \leq y^*_{ij} \\ \text{NA} & \text{if} \ -\gamma_i \leq y^*_{ij} < \gamma_i \\ 0 & \text{if} \quad y^*_{ij} < -\gamma_i \end{cases} \tag{1}$$

If we assume, as in the IRT model, that y^*_{ij} is normally distributed with mean $\beta_j x_i - \alpha_j$ and variance σ^2_j, we can build a probability model for each of the events in Equation 1 (see Section 3.1 below for a more detailed account):

$$\begin{aligned} \Pr(y_{ij} = 1) &= \Phi\left(\beta_j x_i - \alpha_j - \frac{\gamma_i}{\sigma_j}\right) \\ \Pr(y_{ij} = \text{NA}) &= \Phi\left(\frac{\gamma_i}{\sigma_j} - (\beta_j x_i - \alpha_j)\right) - \Phi\left(-\frac{\gamma_i}{\sigma_j} - (\beta_j x_i - \alpha_j)\right) \\ \Pr(y_{ij} = 0) &= 1 - \Phi\left(\beta_j x_i - \alpha_j + \frac{\gamma_i}{\sigma_j}\right) \end{aligned} \tag{2}$$

Figure A represents the relationships in Equation 1 graphically. Though the indifference model adds one γ parameter per legislator, all parameters in the model are still jointly identified as long

Figure A: Illustration of the "indifference" model of *vote choice* and *abstention.* The figure shows the density of $y^*_{ij} \sim \mathcal{N}(\beta_j x_i - \alpha_j, \sigma_j^2)$, which is the latent propensity of legislator i to vote Aye on roll call j. The region around zero represents near indifference between Aye and Nay, and is "observed" as an abstention.



as we fix $\sigma_1 = 1$. Based on Equation 2, we construct the complete-data likelihood function:

$$\mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\phi}|\mathbf{m}, \mathbf{y}) = \prod_{j=1}^{J}\prod_{i=1}^{I} \underbrace{\mathbb{I}(y_{ij}=1)\left[\Phi\left(\beta_j x_i - \alpha_j - \frac{\gamma_i}{\sigma_j}\right)\right]}_{\text{observed Aye}}$$
$$\times \underbrace{\mathbb{I}(y_{ij}=0)\left[1-\Phi\left(\beta_j x_i - \alpha_j + \frac{\gamma_i}{\sigma_j}\right)\right]}_{\text{observed Nay}}$$
$$\times \underbrace{\mathbb{I}(y_{ij}=\text{NA})\left[\Phi\left(\frac{\gamma_i}{\sigma_j} - (\beta_j x_i - \alpha_j)\right) - \Phi\left(-\frac{\gamma_i}{\sigma_j} - (\beta_j x_i - \alpha_j)\right)\right]}_{\text{unobserved vote}} \quad (3)$$

In Equation (3), $\mathbb{I}(a)$ is an indicator function that takes value 1 if a is true and 0 otherwise. Note that the functional form of the indifference model is similar to an ordered probit specification with individual-specific cutpoints.[1] Utility differential y^*_{ij} is a function of $\boldsymbol{\theta} = \{x_i, \alpha_j, \beta_j, \sigma_j\}$. The abstention process is a function of $\boldsymbol{\theta}$ and tolerance threshold γ_i, hence $\boldsymbol{\phi} = \{x_i, \alpha_j, \beta_j, \gamma_i, \sigma_j\}$. Though this non-response mechanism is MAR, the fact that $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$ share common elements means that distinctness cannot be assumed. Since parameters are not distinct, the abstention-generating process is not ignorable and inference should proceed from the complete-data posterior distribution.

As we did for the competing principals model, Table D displays a number of statistics that aid us in evaluating the impact of modeling non-ignorable non-response (please refer to the main text for a detailed explanation of our procedures). Table D confirms our main conclusion, namely, that modeling non-ignorable non-response provides little to no purchase when the pattern of abstentions is relatively low, but leads to more accurate inferences as the incidence of missingness goes beyond 10% or more.

3.1 Derivation of probability model for non-response due to indifference

We show a step-by-step derivation of Equation 2. Suppose legislator i will only register a vote if the the absolute value of his utility differential is larger than his tolerance threshold, i.e., only if $|y^*_{ij}| > \gamma_i$. This assumption allows us to derive the conditions that lead to observation of positive

[1] The cutpoints are identified through a sum-to-zero constraint.

Table D: Full, Nominate, observed-data, and complete-data models for an *indifference* non-response process. Entries correspond to the median and standard deviation of the posterior distribution of each quantity of interest, averaged over 100 legislatures (Nominate standard deviations based on bootstrap routines (Lewis & Poole 2004)).

	Abstention rate: 5%			
	Full	Nominate	Observed	Complete
Bayes residuals	0.147		0.149	0.148
	(0.01)		(0.01)	(0.01)
Pr(correct|seen)	0.776	0.782	0.778	0.777
	(0.02)	(0.02)	(0.02)	(0.02)
Pr(correct|miss)	0.615	0.546	0.548	0.577
	(0.05)	(0.04)	(0.04)	(0.04)
Randall's τ	0.902	0.887	0.9	0.88
	(0.02)	(0.03)	(0.02)	(0.16)
Median Voter	0.269	0.191	0.264	0.259
	(0.2)	(0.1)	(0.2)	(0.19)

	Abstention rate: 10%			
	Full	Nominate	Observed	Complete
Bayes residuals	0.148		0.151	0.149
	(0.01)		(0.01)	(0.01)
Pr(correct|seen)	0.785	0.795	0.791	0.788
	(0.01)	(0.02)	(0.02)	(0.02)
Pr(correct|miss)	0.611	0.529	0.533	0.572
	(0.04)	(0.04)	(0.04)	(0.03)
Randall's τ	0.906	0.886	0.897	0.891
	(0.02)	(0.03)	(0.02)	(0.02)
Median Voter	0.275	0.198	0.255	0.27
	(0.2)	(0.11)	(0.2)	(0.2)

	Abstention rate: 30%			
	Full	Nominate	Observed	Complete
Bayes residuals	0.147		0.178	0.155
	(0.01)		(0.01)	(0.01)
Pr(correct|seen)	0.834	0.876	0.87	0.851
	(0.02)	(0.02)	(0.02)	(0.02)
Pr(correct|miss)	0.614	0.469	0.463	0.562
	(0.02)	(0.03)	(0.02)	(0.03)
Randall's τ	0.902	0.833	0.848	0.859
	(0.03)	(0.04)	(0.03)	(0.03)
Median Voter	0.274	0.143	0.167	0.218
	(0.21)	(0.11)	(0.16)	(0.18)

votes ($y_{ij} = 1$), negative votes ($y_{ij} = 0$), and missing values ($y_{ij} = *$) in the roll-call matrix:

$$y_{ij} = \begin{cases} 1 & \text{if } \gamma_i \leq y^*_{ij} \\ * & \text{if } \gamma_i > y^*_{ij} \geq -\gamma_i \\ 0 & \text{if } \qquad y^*_{ij} < -\gamma_i \end{cases}$$

Note that

$$\begin{aligned} y^*_{ij} &= -(x_i - \zeta_j)^2 + \eta_{ij} - [-(x_i - \psi_j)^2 + \nu_{ij}] \\ &= -x_i^2 + 2x_i\zeta_j - \zeta_j^2 + \eta_{ij} + x_i^2 - 2x_i\psi_j + \psi_j^2 - \nu_{ij} \\ &= 2x_i\zeta_j - \zeta_j^2 + \eta_{ij} - 2x_i\psi_j + \psi_j^2 - \nu_{ij} \\ &= 2(\zeta_j - \psi_j)x_i - (\zeta_j^2 - \psi_j^2) + \eta_{ij} - \nu_{ij} \\ &= \left(\frac{2(\zeta_j - \psi_j)}{\sigma_j} x_i - \frac{\zeta_j^2 - \psi_j^2}{\sigma_j} \right) \sigma_j + \eta_{ij} - \nu_{ij} \\ &= (\beta_j x_i - \alpha_j)\sigma_j + \eta_{ij} - \nu_{ij} \end{aligned}$$

Now, the probability of legislator i recording an aye vote on rollcall j is

$$\begin{aligned} \Pr(y_{ij} = 1) &= \Pr(\gamma_i \leq y^*_{ij}) \\ &= \Pr(\gamma_i \leq (\beta_j x_i - \alpha_j)\sigma_j + \eta_{ij} - \nu_{ij}) \\ &= \Pr(\nu_{ij} - \eta_{ij} \leq (\beta_j x_i - \alpha_j)\sigma_j - \gamma_i) \\ &= \Pr\left(\frac{\nu_{ij} - \eta_{ij}}{\sigma_j} \leq \beta_j x_i - \alpha_j - \frac{\gamma_i}{\sigma_j} \right) \\ &= \Phi\left(\beta_j x_i - \alpha_j - \frac{\gamma_i}{\sigma_j} \right) \end{aligned}$$

The probability of legislator i recording a nay vote on rollcall j is

$$\begin{aligned} \Pr(y_{ij} = 0) &= \Pr(y^*_{ij} < -\gamma_i) \\ &= \Pr((\beta_j x_i - \alpha_j)\sigma_j + \eta_{ij} - \nu_{ij} < -\gamma_i) \\ &= \Pr((\beta_j x_i - \alpha_j)\sigma_j + \gamma_i < \nu_{ij} - \eta_{ij}) \\ &= \Pr\left(\beta_j x_i - \alpha_j + \frac{\gamma_i}{\sigma_j} < \frac{\nu_{ij} - \eta_{ij}}{\sigma_j} \right) \\ &= 1 - \Phi\left(\beta_j x_i - \alpha_j + \frac{\gamma_i}{\sigma_j} \right) \end{aligned}$$

Finally, the probability of legislator i abstaining on rollcall j is

$$\begin{aligned}
\Pr(y_{ij} = *) &= \Pr(-\gamma_i \leq y_{ij}^* < \gamma_i) \\
&= \Pr(-\gamma_i \leq (\beta_j x_i - \alpha_j)\sigma_j + \eta_{ij} - \nu_{ij} < \gamma_i) \\
&= \Pr(-\gamma_i - (\beta_j x_i - \alpha_j)\sigma_j \leq \eta_{ij} - \nu_{ij} < \gamma_i - (\beta_j x_i - \alpha_j)\sigma_j) \\
&= \Pr\left(-\frac{\gamma_i}{\sigma_j} - (\beta_j x_i - \alpha_j) \leq \frac{\eta_{ij} - \nu_{ij}}{\sigma_j} < \frac{\gamma_i}{\sigma_j} - (\beta_j x_i - \alpha_j)\right) \\
&= \Phi\left(\frac{\gamma_i}{\sigma_j} - (\beta_j x_i - \alpha_j)\right) - \Phi\left(-\frac{\gamma_i}{\sigma_j} - (\beta_j x_i - \alpha_j)\right)
\end{aligned}$$

To summarize,

$$\begin{aligned}
\Pr(y_{ij} = 1) &= \Phi\left(\beta_j x_i - \alpha_j - \frac{\gamma_i}{\sigma_j}\right) \\
\Pr(y_{ij} = 0) &= 1 - \Phi\left(\beta_j x_i - \alpha_j + \frac{\gamma_i}{\sigma_j}\right) \\
\Pr(y_{ij} = *) &= \Phi\left(\frac{\gamma_i}{\sigma_j} - (\beta_j x_i - \alpha_j)\right) - \Phi\left(-\frac{\gamma_i}{\sigma_j} - (\beta_j x_i - \alpha_j)\right)
\end{aligned}$$

4 Competing principals models with low separation and abstention in small legislatures

In this section we present information alluded to at the end of "Monte Carlo analysis" in the main paper. Table E presents a summary of inferential gains on small legislatures composed of eleven legislators voting on thirty bills. The main difference is that the average rate of abstention in these legislatures masks important heterogeneity in the voting behavior of legislators: most seldom abstain, and a few abstain heavily. The statistics that we use to gauge inferential gain provide a glimpse of the legislature as a whole, not of individual legislators. Therefore, though we do not see obvious gains, it is quite possible that there are gains in estimates of the ideal points of legislators that abstain heavily; this is a possibility we have not checked.

Table E: Simulation results, small legislatures with a handful of heavy abstainers

Indifference

	Abstention rate: 5%				Abstention rate: 10%			
	Full	Nominate	Observed	Complete	Full	Nominate	Observed	Complete
Bayes residuals	0.123		0.126	0.127	0.12		0.128	0.129
	(0.02)		(0.02)	(0.02)	(0.02)		(0.02)	(0.02)
Pr(correct|seen)	0.813	0.831	0.818	0.813	0.825	0.85	0.835	0.83
	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.03)	(0.04)	(0.04)
Pr(correct|miss)	0.66	0.516	0.513	0.516	0.678	0.524	0.531	0.528
	(0.13)	(0.12)	(0.12)	(0.13)	(0.09)	(0.09)	(0.09)	(0.09)
Randall's τ	0.887	0.797	0.885	0.884	0.87	0.8	0.867	0.872
	(0.07)	(0.09)	(0.06)	(0.06)	(0.07)	(0.11)	(0.08)	(0.06)
Median Voter	0.334	0.181	0.319	0.316	0.347	0.189	0.392	0.387
	(0.22)	(0.08)	(0.21)	(0.22)	(0.23)	(0.07)	(0.23)	(0.24)

Competing principals

	Abstention rate: 5%				Abstention rate: 10%			
	Full	Nominate	Observed	Complete	Full	Nominate	Observed	Complete
Bayes residuals	0.121		0.125	0.122	0.12		0.127	0.123
	(0.02)		(0.02)	(0.02)	(0.02)		(0.02)	(0.02)
Pr(correct|seen)	0.814	0.835	0.819	0.821	0.82	0.842	0.831	0.829
	(0.04)	(0.03)	(0.04)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Pr(correct|miss)	0.717	0.575	0.563	0.602	0.724	0.587	0.577	0.622
	(0.12)	(0.17)	(0.15)	(0.16)	(0.09)	(0.1)	(0.1)	(0.09)
Randall's τ	0.871	0.783	0.86	0.825	0.858	0.795	0.866	0.823
	(0.07)	(0.12)	(0.08)	(0.07)	(0.08)	(0.11)	(0.08)	(0.08)
Median Voter	0.331	0.193	0.309	0.316	0.373	0.177	0.359	0.361
	(0.21)	(0.08)	(0.21)	(0.23)	(0.27)	(0.09)	(0.26)	(0.27)

Table F provides a summary of inferential gains when we analyze legislatures in which the probability of abstention given agreement is only slightly lower than the probability of abstention given disagreement. In these circumstances, the amount of information available to estimate parameters δ is extremely low, and as a consequence the competing principals model improves only marginally on the IRT model, and then only in legislatures with average rates of abstention at 30%.

Table F: Observed- and complete-data models for *competing principals* non-response processes. These runs are based on simulated legislatures with much more homogeneous probabilities of abstention. Entries in these tables correspond to the median and interquartile range of the posterior distribution of each quantity of interest, averaged over 100 legislatures (the last row shows mean and standard deviation of Bayesian residuals).

	Abstention rate: 5%				Abstention rate: 10%				Abstention rate: 30%			
	Full	Nominate	Observed	Complete	Full	Nominate	Observed	Complete	Full	Nominate	Observed	Complete
Bayes residuals	0.147		0.149	0.148	0.148		0.151	0.149	0.147		0.178	0.155
	(0.01)		(0.01)	(0.01)	(0.01)		(0.01)	(0.01)	(0.01)		(0.01)	(0.01)
Pr(correct|seen)	0.776	0.782	0.778	0.777	0.785	0.795	0.791	0.788	0.834	0.876	0.87	0.851
	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Pr(correct|miss)	0.615	0.546	0.548	0.577	0.611	0.529	0.533	0.572	0.614	0.469	0.463	0.562
	(0.05)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.03)	(0.02)	(0.03)	(0.02)	(0.03)
Randall's τ	0.902	0.887	0.9	0.88	0.906	0.886	0.897	0.891	0.902	0.833	0.848	0.859
	(0.02)	(0.03)	(0.02)	(0.16)	(0.02)	(0.03)	(0.02)	(0.02)	(0.03)	(0.04)	(0.03)	(0.03)
Median Voter	0.269	0.191	0.264	0.259	0.275	0.198	0.255	0.27	0.274	0.143	0.167	0.218
	(0.2)	(0.1)	(0.2)	(0.19)	(0.2)	(0.11)	(0.2)	(0.2)	(0.21)	(0.11)	(0.16)	(0.18)

5 Details of simulated legislatures

The simulated ideal points of legislators are drawn from slightly overlapping distributions. For leftist legislators, we use a triangular distribution with support [–2, 0.45] and mode at –1; for rightist legislators, the support is [–0.45, 2] and the mode is 1. Legislators vote on 100 bills with Aye and Nay positions sampled independently, falling roughly within the center of the distribution of legislators' preferences but with most Aye positions (about 80%) constrained to lie to the left of Nay positions, and thus more likely to be preferred by members of the leftist party. We independently sampled two "party ideal points"—i.e., policy preferences for the Left and Right party whips—from distributions centered around the median of leftist and rightist legislators, respectively. For all legislator/item pairs in our simulated set we drew a vote choice—1 or 0 corresponding to Aye or Nay—based on the spatial theoretical reasoning that underlies the IRT model. That is, we assumed that legislators would prefer a proposal if the utility differential between choosing Aye and Nay plus some random error were positive (i.e., $y^*_{ij} + \varepsilon_{ij} > 0$). Random error "flips" about 25% of all vote choices. A vote choice is "flipped" when the utility differential has a different sign than the utility differential plus the random error. Thus, for each legislature we have a full matrix of *known* vote choices $\mathbf{Z}$.

We then produced *observed* vote choice matrices $\mathbf{Y}_{(1)}$ by turning into missing values a given percentage of votes based on the competing principals mechanism of Section 3. We consider three levels of overall non-response—5%, 10%, and 30%—which correspond to commonly-observed rates of missingness in legislatures around the world (see Section 1). We ensured that abstentions were generated according to the competing principals mechanism, but we added random noise to limit the precision of available information. In fact, we expect the IRT specification to perform rather well in recovering legislators' ideal points, if only because we built simulated votes based on the very theoretical assumptions that underlie this model. In short, the observed data $\mathbf{Y}_{(1)}$ are set to be relatively informative about bill and legislator positions, even when we use non-ignorable non-response mechanisms to induce missingness.

Our models are identically identified by stipulating spike priors at -2 and 2 for the (known) most extremist legislators and a $\mathcal{N}(0,4)$ prior on the rest of the legislators. Identification of Nominate models is achieved by fixing the position of the rightmost legislator and constraining all ideal points to the interval [-1,1]. In the competing principals model we stipulate beta(1,1) priors on $\boldsymbol{\delta}$, equivalent to a uniform distribution over the unit range. Due to the small size of legislatures, we draw 5000 burn-in scans and base our inferences on 15,000 further iterations after apparent convergence (thinned every 10 scans). For complete-data models, we monitored convergence of two chains started at different initial values based on Gelman-Rubin's $\widehat{R}$ statistic. For other models, we ran a single chain and monitored convergence based on Geweke's diagnostic statistic. There are a handful of simulated runs that had clearly not converged by iteration 20K; we only consider the first 100 simulated runs for which we could estimate all four models in order to keep comparability.

6 Identification of UN and US senate models

The United Nations in Section 5.1 are identified under rotation by fixing the discrimination parameters of two votes, one in which the US bloc votes favorably against the Soviet bloc, and one in which the Soviet bloc votes favorably against the US bloc. To identify scale, priors on ideal points are standard normal. Our prior distributions for all δ parameters are "flat" Beta(1,1) distributions, which give equal probability to all values in the unit range. We run two chains for each model for 100K scans after 50k burn-in, and use a thinned sample of 1,000 scans to describe posterior distributions. We assess convergence based on the Geweke and Gelman-Rubin $\hat{R}$ statistics and on plots of the posterior distributions of parameters.

Similarly, to ensure comparability of the observed- and complete-data models in Section 5.2, we use the exact same narrow prior distributions on two bills, one where a majority of Democrats rolls a majority of Republicans and one where a majority of Republicans rolls a majority of Democrats. Our inferences are based on three Markov chains, with 25k burn-in and 50k scans thinned every 50^{th} draw. Convergence diagnostics include graphical inspections of posterior distributions of all parameters, Geweke statistics, and Gelman-Rubin $\hat{R}$ statistics.

7 Jags code for all models

The Jags code in this section builds MCMC samplers for all the models discussed in the paper.

7.1 IRT model

```
model {
  for(i in 1:n.legislators) {
    for(j in 1:n.item) {
      rc[i,j] ~ dbern(p[i,j]);
      probit(p[i,j]) <- mu[i,j];
      mu[i,j] <- beta[j]*theta[i] - alpha[j];
    }
  }
  # PRIORS
  for(j in 1:n.item) { alpha[j] ~ dnorm(0, 0.25); }
  for(j in 1:n.item) { beta[j]  ~ dnorm(0, 0.25); }
  theta[1] <- -2;                  # spike leftmost legislator
  for(i in 2:(n.legislators-1)) { theta[i] ~ dnorm(0, 1); }
  theta[n.legislators] <- 2;      # spike rightmost legislator
}
```

7.2 Indifference model

```
model {
  for (i in 1:n.legislators) {
    for (j in 1:n.item) {
      y[i,j] ~ dcat(p[i,j,1:3]);
      p[i,j,1] <- 1 - Q[i,j,1];
      p[i,j,2] <- Q[i,j,1] - Q[i,j,2];
      p[i,j,3] <- Q[i,j,2];
      probit(Q[i,j,1]) <- beta[j]*theta[i] - alpha[j] + gamma[i]/sigma[j];
      probit(Q[i,j,2]) <- beta[j]*theta[i] - alpha[j] - gamma[i]/sigma[j];
    }
  }
  # PRIORS
  for(j in 1:n.item) { alpha[j] ~ dnorm(0, 0.25); }
  for(j in 1:n.item) { beta[j]  ~ dnorm(0, 0.25); }
  sigma[1] <- 1;
  for(j in 2:n.item) { sigma[j] ~ dexp(0.1); }
  theta[1] <- -2;
  for(i in 2:(n.legislators-1)) { theta[i] ~ dnorm(0, 1); }
  theta[n.legislators] <- 2;
  for (i in 1:n.legislators) { gamma[i,2] ~ dexp(0.1); }
}
```

7.3 Competing principals model

```
model {
  for (i in 1:n.legislators) {
    for (j in 1:n.item) {
```

```
      y[i,j] ~ dcat(Q[i,j,1:3]);  # code Nay=1, NA=2, Aye=3
      Q[i,j,1] <- pow(probVoteDisagree*(1-IRT[i,j]), p[i,j])
                  * pow(probVoteAgree*(1-IRT[i,j]), (1 - p[i,j]));
      Q[i,j,2] <- 1 - Q[i,j,1] - Q[i,j,3];
      Q[i,j,3] <- pow(probVoteAgree*IRT[i,j], p[i,j])
                  * pow(probVoteDisagree*IRT[i,j], (1 - p[i,j]));
      probit(IRT[i,j]) <- beta[j]*theta[i] - alpha[j];
      p[i,j] <- lead[party.membership[i],j];
    }
  }
  # PRIORS
  pi[1] <- 1-delta[1] #Pr(Abstain|Disagree)=1-Pr(Vote|Disagree)
  pi[2] <- 1-delta[2] #Pr(Abstain|Agree)=1-Pr(Vote|Agree)
  # prior on Pr(Vote|Disagree) is a bimodal distribution with
  # most probability mass close to either 0 or 1
  delta[1] <- probVoteDisagree
  logit(probVoteDisagree) <- prior.prob[1];
  # prior on Pr(Vote|Agree) forces Pr(Vote|Agree)>Pr(Vote|Disagree)
  delta[2] <- probVoteAgree
  logit(probVoteAgree) <- prior.prob[1]+prior.prob[2];
  prior.prob[1] ~ dnorm(0,0.1);
  prior.prob[2] ~ dexp(0.1);
  for(j in 1:n.item) { alpha[j]  ~ dnorm(0, 0.25); }
  for(j in 1:n.item) { beta[j] ~ dnorm(0, 0.25); }
  theta[1] <- -2;              # spike leftmost legislator
  for(i in 2:(n.legislators-1)) { theta[i] ~ dnorm(0, 0.25); }
  theta[n.legislators] <- 2;    # spike rightmost legislator
}
```

7.4 Competing principals model for United Nations General Assembly

```
model {
  for (i in 1:n.legislators) {
  for (j in 1:n.item) {
    probit(CJR[i,j]) <- beta[j]*theta[i] - alpha[j];
    }
  }
  for (k in 1:2) {
    for (j in 1:n.item) {
      lead[k,j] ~ dbern (CJR[principal[k],j])
    }
  }
  for (i in 1:n.partisans) {
    for (j in 1:n.item) {
      Y[i,j] ~ dcat(Q[i,j,1:3]);    # code Nay as 1, NA as 2, Aye as 3
      Q[i,j,1] <- pow(probVoteDisagree[party.membership[i]] *
                  (1-CJR[i,j]), p[i,j]) *
                  pow(probVoteAgree[party.membership[i]] *
                  (1-CJR[i,j]), (1 - p[i,j]));
```

```
      Q[i,j,2] <- 1 - Q[i,j,1] - Q[i,j,3];
      Q[i,j,3] <- pow(probVoteAgree[party.membership[i]] *
                  CJR[i,j], p[i,j]) *
                  pow(probVoteDisagree[party.membership[i]] *
                  CJR[i,j], (1 - p[i,j]));
      p[i,j] <- lead[party.membership[i],j];
    }
  }
  for (i in (n.partisans+1):n.legislators) {
    for (j in 1:n.item) {
      Y[i,j] ~ dbern(CJR[i,j]);
    }
  }
# PRIORS
  # prior on probVoteAgree and probVoteDisagree for both parties
  # are uniform distributions over the unit-range
  for (m in 1:4) {
    delta[m] ~ dbeta(1,1);
  }
  probVoteDisagree[1] <- delta[1];
  probVoteAgree[1]    <- delta[2];
  probVoteDisagree[2] <- delta[3];
  probVoteAgree[2]    <- delta[4];
  for(j in 1:n.item) { alpha[j] ~ dnorm(0, 0.25); }
   beta[fix.bill[1]] <-  4
  for(j in (fix.bill[1]+1):(fix.bill[2]-1)) { beta[j]  ~ dnorm(0, 0.25); }
   beta[fix.bill[2]] <- -4
  for(j in (fix.bill[2]+1):n.item) { beta[j]  ~ dnorm(0, 0.25); }
  for(i in 1:n.legislators)  { theta[i] ~ dnorm(0, 1); }
}
```

7.5 Competing principals model for US Senate

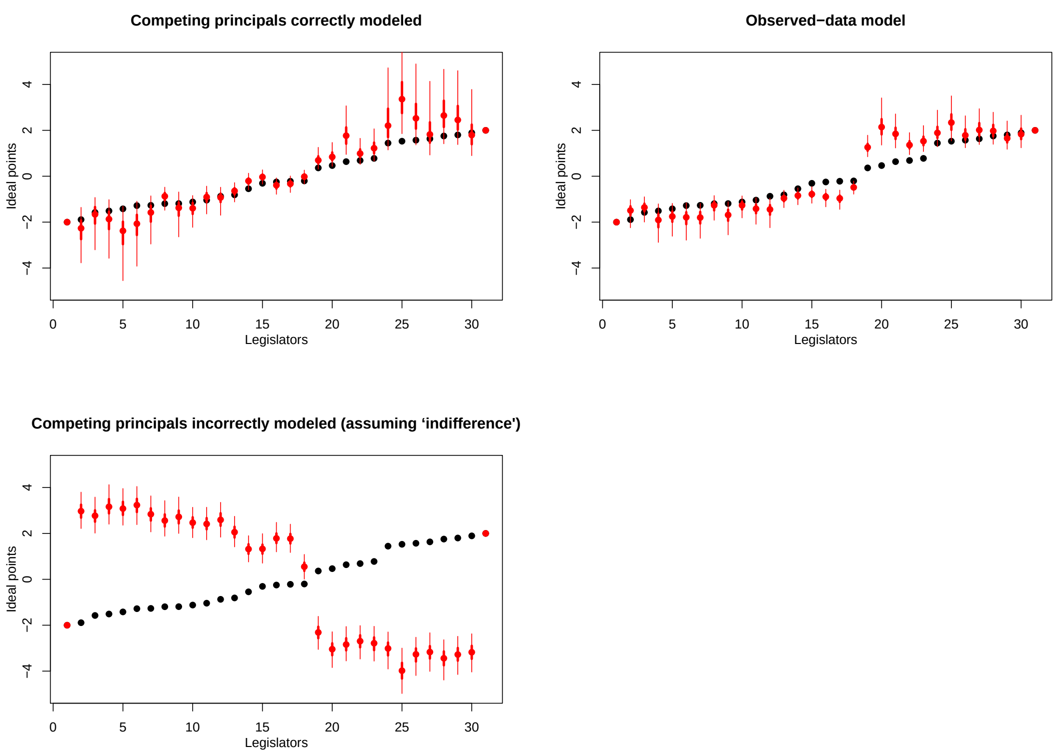
```
model {
  for (i in 1:(n.legislators-4)) {  # All Senators except presidential candidates
   for (j in 1:n.item) {
     y[i,j] ~ dbern(mu[i,j]);
     probit(mu[i,j]) <- beta[j]*theta[i] - alpha[j];
    }
  }
  for (i in (n.legislators-3):n.legislators) {   # Four presidential candidates
    for (j in 1:n.item) {
      probit(CJR[i,j]) <- beta[j]*theta[i] - alpha[j];
      y[i,j] ~ dcat(Q[i,j,1:3]);         # code Nay as 1, NA as 2, Aye as 3
      Q[i,j,1] <- pow(probVoteDisagree*(1-CJR[i,j]), p[i,j])
                  * pow(probVoteAgree*(1-CJR[i,j]), (1-p[i,j]));
      Q[i,j,2] <- 1 - Q[i,j,1] - Q[i,j,3];
      Q[i,j,3] <- pow(probVoteAgree*CJR[i,j], p[i,j])
                  * pow(probVoteDisagree*CJR[i,j], (1 - p[i,j]));
```

```
      p[i,j] <- lead[party.membership[i],j];
    }
  }
# PRIORS
  # prior on probVoteAgree and probVoteDisagree
  # are uniform distributions over the unit-range
  probVoteDisagree ~ dbeta(1,1)  # Flat prior
  probVoteAgree ~ dbeta(1,1)     # Flat prior
  # Comment out the following lines to use informative priros
  # on probVoteAgree and probVoteDisagree
  # probVoteDisagree ~ dbeta(6,28)   # Informative prior
  # probVoteAgree ~ dbeta(16,35)     # Informative prior
for(j in 1:n.item) { alpha[j] ~ dnorm(0, 0.25); }
for(j in 1:(fix.bill[1]-1)) { beta[j]  ~ dnorm(0, 0.25); }
    beta[fix.bill[1]] <- 2
for(j in (fix.bill[1]+1):(fix.bill[2]-1)) { beta[j]  ~ dnorm(0, 0.25); }
    beta[fix.bill[2]] <- -2
for(j in (fix.bill[2]+1):n.item) { beta[j]  ~ dnorm(0, 0.25); }
for(i in 1:n.legislators)  { theta[i] ~ dnorm(0, 1); }
}
```

8 Illustrating the consequences of applying an incorrect complete-data model

We emphasize here one of the main lessons from the paper: The competing principals model is not a "one size fits all" estimation procedure to deal with every single instance of suspected non-ignorable non-response. To illustrate this point, we look at one simulated legislature with high abstention rate and abstentions generated by a competing principals dilemma. In Figure B, we plot three graphics, each recovering estimates of "true" ideal points based on a different model. In the leftmost plot, we use the competing principals model of Equation 3 to recover ideal points. This is the "correct" model, given that we produced abstentions by following a competing principals logic. It is obvious that the competing principals complete-data model does a very good job of determining the relative positions of true ideal points; note also that credible intervals are adequately large, reflecting uncertainty from unobserved values that are assumed missing not at random. In the center plot, we estimate the observed-data model, that is, the canonical Clinton-Jackman-Rivers IRT specification. This model yields inferences that are slightly incorrect, particularly for centrist legislators, and produces credible intervals for these inferences that are too optimistic. Note, however, that the canonical IRT specification that assumes MAR abstentions produces better inferences than those show in the rightmost plot. In that plot, we incorrectly fit a model that assumes that abstentions were generated by the indifference mechanism shown in Sections 3 and 3.1 of this appendix. The recovered order of ideal points is entirely wrong; to add insult to injury, the credible intervals for these inferences are very tight, suggesting an extreme amount of certainty in the relative order of these ideal points.

Figure B: Ideal point estimates based on three different models. Black dots correspond to the "true" ideal points of legislators. The red dots and lines correspond to medians (and 75%/95% highest posterior density intervals) of the posterior distributions of $\boldsymbol{\theta}$.



9 A comparison of the competing principals model and an ordered-outcomes model for UN-GA votes

Voeten (2004) studies a subset of UN General-Assembly votes, namely, those that were of such importance for the United States that its diplomatic corps expended effort trying to secure a favorable outcome. In this context, Voeten considers "explicit abstentions" as signaling a weaker preference for a Nay vote. In other words, he considers Aye/Abstention/Nay as an ordered outcome and essentially builds an ordered logit item-response theory model to analyze these data. Voeten's model is slightly complicated by the explicit inclusion of country-level information through a hierarchical model for the ideal point parameters that capture the policy preferences of voting countries. In the interest of comparability, we ignored this wrinkle and coded the following MCMC sampler, which captures the relevant ordered-outcome structure of Voeten's model.

```
model {
  for (i in 1:n.legislators) {
    for (j in 1:n.item) {
    # Categorical outcome: 1=NAY, 2=Abstain, 3=AYE
       y[i,j] ~ dcat(p[i,j,1:3]);
       p[i,j,1] <- 1 - Q[i,j,1];
       p[i,j,2] <- Q[i,j,1] - Q[i,j,2];
       p[i,j,3] <- Q[i,j,2];
     # Linear predictor: Cutpoints are item-specific
     # Item-specific alpha and tau parameters are identified through
     # sum-to-zero constraint
      for (i.cut in 1:n.cut){
        logit(Q[i,j,i.cut]) <- beta[j]*theta[i] + alpha[j] - tau[j,i.cut];
      }
    }
  }
  # PRIORS
  for(j in 1:n.item) {
    alpha[j] ~ dnorm(0, 0.25);
    # Sum-to-zero constraint to identify item parameters
    for(i.cut in 1:n.cut){
      tau.unsorted[j,i.cut] ~ dnorm(alpha[j],0.01)
    }
      tau[j,1:n.cut] <- sort(tau.unsorted[j,1:n.cut]
                             -mean(tau.unsorted[j,1:n.cut]))
  }
  # Model identification under rotation uses priors identical to our model
  beta[fix.bill[1]] <- 4
  for(j in (fix.bill[1]+1):(fix.bill[2]-1)) { beta[j]  ~ dnorm(0, 0.25); }
  beta[fix.bill[2]] <- -4
  for(j in (fix.bill[2]+1):n.item) { beta[j]  ~ dnorm(0, 0.25); }
  for(i in 1:n.legislators) { theta[i] ~ dnorm(0,1); }
}
```

As we apply this model to votes from the General-Assembly in 1970, we underscore two points: First, we look at *all* votes cast by the General-Assembly that year, not only those that were of particular concern to the United States. Second, our competing principals logic implies that *all* missing votes can in principle be strategic; this applies to abstentions, absences and instances of present but did not vote. Regarding this last point, we consider here a version of Voeten's model where *all* missing values are coded as abstentions; Voeten himself distinguishes between abstentions and absences.[2]

[2]As far as we can tell, Voeten considers "absences" and instances of "present but did not vote" as MAR and

A comparison of the inferred ideal positions of General-Assembly countries based on these two models appears in Figure C. There are some obvious differences between these models, especially in the location of Portugal, South Africa, and Great Britain (countries generally aligned with the United States) and Yemen and Albania in the Soviet era of influence. Of particular note is the location of Romania and Cuba vis-à-vis the rest of the countries in the Soviet camp. In the competing principals model, these two countries are occupy a space distinct from that of the rest of the Soviet camp; the same distinction is evident in Voeten's model, though the distance bewteen Romania-Cuba and the rest of the bloc is larger.

Which model is correct? Both relative orderings seem plausible. As we mention in the paper, a comparison of deviance statistics between our competing principals model and the canonical IRT model is not possible because the outcomes are codified in a different way (a dichotomous indicator with potential missing values for the IRT model versus a trichotomous indicator without missing values for the competing principals complete-data model). However, such comparison becomes possible when pairing Voeten's and our model; we add that the comparison is only strictly possible when we consider *all* missing values as abstentions, which is what we have done here. We estimate the mean deviance statistic for the competing principals complete-data model as 4466, whereas our estimate of this statistic for Voeten's ordered-outcomes model is 9497. Unarguably, the fit of our model is much better than the fit of Voeten's model. However, we plead once more to keep in mind the caveats to the use of Voeten's model, especially the fact that he purports to use this model to study a very specific subset of General Assembly votes.

imputes these values using the Chib-Albert data augmentation mechanism that underlies the canonical Clinton-Jackman-Rivers IRT model.

Figure C: Comparison of the competing principals model and an ordered-outcomes model for UN-General Assembly votes