The B.E. Journal of Theoretical Economics

Advances

Volume 10, *Issue* 1	2010	*Article* 6

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Recommended Citation

Zvika Neeman and Nir Vulkan (2010) "Markets versus Negotiations: The Predominance of Centralized Markets," *The B.E. Journal of Theoretical Economics*: Vol. 10: Iss. 1 (Advances), Article 6.

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Abstract

The paper considers the consequences of competition between two widely used exchange mechanisms, a "decentralized bargaining" market, and a "centralized" market. In every period, members of a large heterogenous group of privately-informed traders who each wish to buy or sell one unit of some homogenous good may opt for trading through one exchange mechanism. Traders may also postpone their trade to a future period. It is shown that trade outside the centralized market completely unravels. In every strong Nash equilibrium, all trade takes place in the centralized market. No trade ever occurs through direct negotiations.

KEYWORDS: centralized markets, decentralized markets, decentralized bargaining, market microstructure, competition

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*We thank Alon Klement, Tamar Kugler, Muriel Niederle, Gerhard Orosel, Mark Satterthwaite, and Asher Wolinsky for useful discussions. Itzhak Gilboa provided especially valuable comments and suggestions. We also thank seminar participants at the NBER-Decentralization conference (Fall, 2000), BU, Cornell, Copenhagen, Haifa, Hebrew, Helsinky, Oxford, Tel-Aviv, and Vienna, an editor, and two anonymous referees, for their comments. The first draft of this paper was written while Neeman was visiting the Centre for Economic Learning and Social Evolution (ELSE) in University College London. We thank ELSE for their hospitality. Neeman is grateful for the generous financial support of the NSF under grant SBR-9806832. Vulkan is grateful for the generous financial support from EPSRC award GR MO7052.

1. Introduction

This paper considers the consequences of competition between two widely used exchange mechanisms, a "decentralized bargaining" market, and a "centralized" market. Competition assumes the following form: in every period, surviving and new members of a large heterogenous group of privately-informed traders who each wish to buy or sell one unit of some homogenous good may opt for trading through either (1) direct negotiations with other buyers and sellers (a decentralized bargaining market), or (2) a centralized market. If they so wish, traders may also postpone their trade to a future period.

A *decentralized bargaining market* is an idealization of what takes place in a bazaar, or a Middle-Eastern Suq. Buyers and sellers are matched with each other and bargain over the terms of trade. If an agreement is reached, the traders leave the market. Otherwise, they return to the general pool of traders, are possibly re-matched with another trader, and so on. Importantly, transaction prices in such a market typically vary across the different matches depending, among other things, on the individual traders' costs and willingness to pay. Consequently, decentralized bargaining is characterized by the fact that different traders may transact at *different* prices at the same time. In contrast, a *centralized market* is a form of exchange with a single price and centralized clearing. It is characterized by the fact that in any point in time, all those traders who transact do so at the *same* price. Examples include a sealed-bid double-auction and a *call market*.[1]

The study of the outcome of such a competition is of interest for two reasons. First, the question of what form of exchange is likely to attract large volumes of trade is an important theoretical and practical problem. To answer this question, it is not enough to analyze the properties of different exchange mechanisms in isolation. Because traders' choices of where to trade are endogenous, the very existence of a competing exchange mechanism may affect the outcome in any given mechanism. In other words, the question is what kind of exchange mechanisms will flourish when traders are free to choose the exchange mechanism through which to transact. Obviously, because exchange mechanisms that may initially be attractive to sellers may be shunned by buyers and vice-versa, mechanisms that generate large volumes of trade must be sufficiently attractive to *both* buyers and sellers. Second, although

[1] Call markets are used, among other things, to determine the daily opening prices of the stocks listed in the New York Stock Exchange, and to fix copper and gold price in London (Schwartz, 1988).

the description of the two competing exchange mechanisms in this paper is extremely stylized, the comparison is between a "traditional" and a "modern" form of exchange. Understanding the forces that determine the consequences of such a competition may shed some light on the development of actual market mechanisms.

The main result of this paper is that under fairly general conditions, in every strong Nash equilibrium, all trade is conducted through the centralized market – no opportunities for mutually advantageous trade exist outside the centralized marketplace. Obviously, because traders cannot trade alone, there also exists an equilibrium in which all traders trade only through direct negotiations, however, unlike an equilibrium where all trade takes place in the centralized market, such an equilibrium is not coalition proof, and moreover, it can be destabilized by the joint deviation of a relatively small coalition of traders.

The approach in this paper is distinguished by the fact that, in contrast to standard models that impose assumptions about traders' *behavior* and then derive the implications of these assumptions with respect to market structure, here the assumptions are imposed directly on the distribution of transaction prices under the two competing mechanisms. This "reduced-form" approach allows us to bypass the main difficulty associated with the standard approach, namely the characterization of equilibrium properties, which is intractable in all but the simplest models. In contrast, the approach followed in this paper permits the consideration of a general dynamic setup with heterogenous traders who have private information about their willingness to pay and cost and face aggregate uncertainty.

There is a vast literature on the microstructure of markets and trading institutions. This literature can be divided into several broad categories. First, there is a large literature that has confined its attention to the analysis of different market mechanisms in isolation. In this literature, comparisons between different market mechanisms are usually done from the perspective of the seller, asking what mechanism a single seller would prefer under the assumption that buyers have no choice but to participate in the chosen mechanism (as in, e.g., Milgrom and Weber, 1982). A second category, into which this paper belongs, consists of papers that consider the case in which traders choose through which one of a small number of given mechanisms to conduct their trades (see, e.g., Gehrig, 1993; Rust and Hall, 2003, and the references therein). A number of papers permit traders to choose from a large number of possible trade mechanisms (see, e.g., McAfee, 1993; Peters, 1994; and subsequent literature) but in models where competing sellers choose a type of auction through which to sell and buyers choose in which seller's auction to participate. Finally, there exists a voluminous related literature in finance that emphasizes the importance of transaction costs, information, adverse selection, and transparency, but that pays less attention to the strategic issues considered here (for a recent survey of this literature, see Madhavan, 2000).

The paper that is most closely related to ours is Rust and Hall (2003) who consider a stationary environment in which in every period buyers and sellers can choose between trading through "middlemen" or a "specialist" who each set their own bid-ask spreads. Trading through a middleman and through the specialist in their model is similar to trading through direct negotiations and a centralized market in our model. But in Rust and Hall's model, a trader who wants to trade through a middleman has to wait one period. This implies that the specialist can operate even if it is less efficient than some of the middlemen and that the bid-ask spread in the centralized market is larger than that of any of the middlemen. It therefore follows that "eager" traders would prefer to trade through the centralized market (because the difference in price is not worth the waiting cost) whereas "less eager" traders would prefer trading through the middlemen (they would lose if they trade through the centralized market, and gain if they trade through a middleman, and a discounted gain is still postive and so better than a loss). This pattern is similar to the one that is described in this paper, but, importantly, it is due to a different reason. As explained below, in this paper, eager traders prefer the centralized market because trading there doesn't move the price against them as much as in the decentralized market. This effect is absent in Rust and Hall's model. Furthermore, even as trading costs converge to zero, the specialist in Rust and Hall's model would still charge a positive bid-ask spread to maximize its profit, which implies that, unlike in our model, the unraveling of the decentralized market in Rust and Hall's model would never be complete. In addition, Rust and Hall employ a very specific model of the centralized and decentralized market whereas as explained below our "reduced form approach" admits a wide range of trading institutions.

The rest of the paper proceeds as follows. In the next section, we describe a simple example that illustrates our main insight. In Section 3, we present the general model and the details of modelling the centralized market and direct negotiations. Analysis of the model is presented in Section 4, and a few concluding remarks are offered in Section 5.

2. An Example

We describe a simple static example that provides an intuition for our main result. The general model is dynamic, has a large number of heterogenous traders, and unlike the example, which employs a specific model of direct negotiations, is consistent with many different models of direct negotiations.

Consider an environment with six traders, three buyers and three sellers. Each buyer wants to buy, and each seller wants to sell, one unit of some homogenous good. The buyers' willingness to pay for the good are 10, 9, and 2, respectively; and the sellers' costs of producing the good are 0, 1, and 8, respectively. Note that the static nature of the example implies that there is no reason to abstain from trade.

If all the traders trade in a centralized market where they behave as price-takers, then any price $p \in [2, 8]$ can serve as a market clearing price. Suppose, for simplicity, that the price that prevails in the centralized market is $p = 5$. The two buyer's types with the willingness to pay of 10 and 9 trade with the two seller's types whose costs are 0 and 1. The payoff to the buyer whose willingness to pay is 10 and to the seller whose cost is 0 is 5; and the payoff to the buyer whose willingness to pay is 9 and to the seller whose cost is 1 is 4. The other buyer's and seller's types do not trade in the centralized market, and obtain, each, a payoff of 0.

Suppose on the other hand that the traders engage in direct negotiations with each other. Suppose further that this negotiation assumes the following form: a first stage of random matching between the buyers and sellers, followed by a second stage of split-the-surplus bargaining. In this case, the expected payoff to the buyer whose willingness to pay is 10 is 3.5 because with probability $\frac{1}{3}$ this buyer is matched with the seller whose cost is 8, trades at the price 9, and obtains a payoff of 1, with probability $\frac{1}{3}$, it is matched with the seller whose cost is 1, trades at the price $\frac{11}{2}$, and obtains a payoff of $\frac{9}{2}$, and with probability $\frac{1}{3}$, it is matched with the seller whose cost is 0, trades at the price 5, and obtains a payoff of 5. Similarly, the expected payoff to the buyer whose willingness to pay is 9 is 3, and the expected payoff to the buyer whose willingness to pay is 2 is $\frac{1}{2}$ because when this buyer is matched with the seller whose cost is 8, no trade takes place. Similarly, the expected payoff to the seller whose cost is 8 is $\frac{1}{2}$ and the expected payoffs to the sellers whose costs are 0 and 1 are 3.5 and 3, respectively.

Obviously, the two buyer's types with the high willingness to pay and the two seller's types with the low costs (the relatively more "eager" types) are better off in the centralized market compared to direct negotiations. Even if they alone switch to trading through the centralized market, they are still better off as they can still trade at the competitive equilibrium price $p = 5$. However, once they switch, the remaining buyer and seller become worse off since they lose the ability to trade. They, too, may switch to the centralized market, but this will not improve their situation, because they do not get to trade in the centralized market either.

Intuitively, what makes the centralized market more attractive to the two buyer's types with the high willingness to pay and the two seller's types with the low cost is that, relative to direct negotiations, the extent to which their high willingness to pay and low cost is translated into higher prices paid and lower prices received, respectively, is smaller. Consequently, the eager types of the buyer and seller are led into trading in the centralized market, which in turn, leads to the unraveling of trade through direct negotiations.

In this simple example, it is easy to imagine a direct negotiation procedure that would lead to exactly the same outcome as the centralized market (for example, by changing the matching process to one that ensures that the buyer with willingness to pay 10 is matched with the seller whose cost is 0, and the buyer with willingness to pay 9 is matched with the seller whose cost is 1). Obviously, in such a case we cannot obtain the result that all trade must take place in the centralized market. The literature on decentralized bargaining (surveyed in Osborne and Rubinstein, 1990, and discussed in more detail at the end of section 3.2) has devoted much attention to the question of how likely is "frictionless" decentralized bargaining to give rise to the centralized market (Walrasian) outcome and has concluded that the required conditions are very strong. In any case, the assumptions imposed below preclude this possibility.

Finally, it is important to emphasize that what makes the centralized market more attractive to eager traders is not just the fact that it offers a more efficient form of matching than direct negotiations, but also the way in which the surplus from trade is distributed among the different traders' types. More specifically, the distribution of surplus in the centralized market is biased in favor of more eager traders, which is what starts the process of unraveling. This can be best seen by comparing the expected payoff to different traders' types under the centralized market and under direct negotiations with efficient matching. Suppose for example that direct negotiation assumes the following form: a first stage of matching in which with probability $\frac{1}{2}$ the buyers with willingness to pay 10, 9, and 2 are matched with the sellers whose costs are

0, 1, and 8, respectively; and with probability $\frac{1}{2}$ the buyers with willingness to pay 10, 9, and 2 are matched with the sellers whose costs are 1, 0, and 8, respectively. Under this direct negotiation procedure, the expected payoffs to the buyers with willingness to pay 10, 9, and 0, and to the sellers with costs 0, 1, and 8, are 4.75, 4.25, and 0, respectively. The buyer with willingness to pay 9 and the seller with cost 1 are better off with this type of direct negotiations than under the centralized market, but the buyer with willingness to pay 10 and the seller with cost 0, or the most eager traders, are worse off, and once they switch into trading in the centralized market, the other types would be better off following them there.[2]

3. The Model

We consider a dynamic model with a large number of buyers and sellers of some discrete homogenous good. Time is also discrete and is indexed by $t \in \{1, 2, ...\}$. In each period, each seller has one unit to sell, and each buyer is interested in buying one unit.[3] Traders are characterized by their types: their willingness to pay for one unit of the good if buyers, and their cost of producing one unit if sellers. It is commonly known that sellers' costs and buyers' willingness to pay are stochastically independent and are drawn from the grid $\mathcal{G} = \left\{0, \frac{1}{K}, \frac{2}{K}, ..., \frac{K-1}{K}, 1\right\}$ where K is some large integer. "Eager" buyer types have high willingness to pay, and "eager" seller types have low costs.[4]

We assume that in every period t, N_t^B new buyers and N_t^S new sellers appear; the numbers of traders N_t^B and N_t^S may be stochastic, but we assume that they are "large" and independent of traders' types. The cumulative distributions of the new buyers' and sellers' types in every period need not be symmetric or identical across buyers and sellers. In addition to the new traders, the group of traders in any period t may also include traders that have

[2] The insight that the division of the gains from trade in bargaining may make bargaining less stable relative to an auction has been noted by Lu and McAfee (1996) in the context of a simpler model with homogenous buyers and sellers.

[3] This implies no loss of generality compared with the assumption that traders are each interested in trading a finite number of units of the good. Traders that are interested in trading $k < \infty$ units of the good can be treated as k different traders.

[4] These types are called eager because their high willingness to pay and low cost, respectively, implies that they can, and are therefore also more likely to, trade profitably at a wider range of prices.

appeared in the previous period but who for some reason did not trade. Every trader knows its own type but may be uncertain about the number of other traders and their types. Note that the fact that N_t^B and N_t^S are stochastic implies that the model admits aggregate uncertainty.

Every period, buyers and sellers may either attempt to trade through a centralized market or through direct negotiations. Traders may also refrain from trade and wait for the next period. A trader who for some reason does not trade in any given period re-appears in the next period (with the same type) with some probability $0 \leq \delta < 1$, which may vary across the different traders. The different parameters δ may also be interpreted as the traders' discount factor. Buyers and sellers are assumed to be (risk neutral) expected utility maximizers. A buyer with a willingness to pay v who transacts at the price p, τ periods after it first appeared in the market, obtains the payoff $\delta^{\tau}(v-p)$, and a seller with cost c who transacts at the price p, τ periods after it first appeared in the market, obtains the payoff $\delta^{\tau}(p-c)$. Traders who disappear without trading, or who never trade, obtain a payoff of zero.

In every period $t \geq 1$, the traders' choices about whether to attempt to trade through the centralized market or through direct negotiations may depend on their own type, and on the history of trade in the centralized market and direct negotiations, respectively. Obviously, traders' choices may also depend on their beliefs about what other traders, "new" and "old," would do. A Nash equilibrium is a sequence of profiles of traders' choices of if and where to trade, such that each trader's choice is optimal given other traders' choices. To simplify the analysis, we assume that traders who are indifferent between trading in the centralized market and direct negotiation in some period t, opt for trading, if at all, through direct negotiations.

We describe the details of trade in the centralized market and through direct negotiations in the next two subsections.

3.1. Centralized Markets

For our purposes, a centralized market may be idealized as follows: in every period $t \in \{1, 2, ...\}$, each buyer and each seller who opts for the centralized market specifies a bid and an ask price, respectively. Trade takes place at a Walrasian (market-clearing) price, denoted p_t^M, between the buyers whose bids are higher than or equal to this centralized market price, and the sellers whose asks are lower than or equal to the centralized market price. In case of a shortage or a surplus, the allocation is carried out as far as possible by assigning priority to sellers whose asks are the smallest and buyers whose bids are the largest. If this does not complete the allocation, then a fair lottery determines which of the remaining traders on the long side of the market trade.

We assume that the aggregate uncertainty in the economy is such that the centralized market price p_t^M is (at least) a little volatile, and that the support of the buyers' and sellers' beliefs about the distribution of the price overlap.

Traders may recognize their ability to influence the centralized market price in their favor. They may do so by submitting ask prices that are higher than their true costs if they are sellers, and by submitting bids that are lower than their true willingness to pay if they are buyers. We denote the ask price quoted by a seller with cost c by $a(c)$ and the bid made by a buyer with willingness to pay v by $b(v)$. We assume that $a(c)$ and $b(v)$ are monotone nondecreasing in c and v, respectively.

For every $c, v \in [0, 1]$, denote the expected centralized market price, taking into account the possibility of manipulation, as perceived by a seller with cost c and by a buyer with willingness to pay v by $E\left[p_t^M |c\right]$ and $E\left[p_t^M |v\right]$, respectively.

We make an important assumption about what happens in the centralized market as the number of traders who opt to trade there increases, namely:

Convergence of Beliefs. We assume that the difference

$$E\left[p_t^M |v = 1\right] - E\left[p_t^M |c = 0\right]$$

decreases to zero as the number of buyers and sellers who opt for the centralized market increases.

To justify this assumption, note that a buyer with willingness to pay one has the same beliefs as a seller with cost zero about the distribution of other traders' costs and willingness to pay. This implies that the difference in their beliefs about the centralized market price is due to the fact that: (i) while the seller believes that the buyer with willigness to pay one has a randomly drawn willigness to pay, the buyer knows that its willigness to pay is one; and (ii) while the buyer believes that the seller with cost zero has a randomly drawn cost, the seller knows that its cost is zero. The fact that the Walrasian price is nondecreasing in the buyers' bids and the sellers' ask prices (and the assumption that the latter are monotone nondecreasing in their willingness to pay and cost, respectively) implies that $E\left[p_t^M |v = 1\right] \geq E\left[p_t^M |c = 0\right]$. As the number of traders in the market increases, the effect that any single trader has on the Walrasian price decreases, and so the difference $E\left[p_t^M \,|v=1\right] - E\left[p_t^M \,|c=0\right]$ decreases to zero.

Convergence of beliefs, the assumption that the centralized market price is volatile and that buyers' and sellers' beliefs about the distribution of the price overlap are consistent with the description of a competitive market with privately informed traders in Rustichini, Satterthwaite, and Williams (1994) who, under the assumption that the number of traders in the market is exogenously given, derived convergence of beliefs as a result. It is also consistent with the more recent papers by Cripps and Swinkels (2006) and Reny and Perry (2006) who have considered market equilibrium under somewhat more general assumptions.[5]

Let $Q_t(c)(\cdot)$ denote the distribution of the centralized market price as perceived by a seller with cost c. If this seller behaves as a price-taker, then its expected payoff from opting for the centralized market in period t is given by[6]

$$S_t^M(c) = \int_c^1 (p-c)\, dQ_t(c)(p).$$

Similarly, let $Q_t(v)(\cdot)$ denote the distribution of the centralized market price as perceived by a buyer with willingness to pay v. If this buyer behaves as a price-taker, then its expected payoff from opting for the centralized market in period t is given by

$$B_t^M(v) = \int_0^v (v-p)\, dQ_t(v)(p)$$

Lemma 1. *For every willingness to pay and cost $v, c \in \mathcal{G}$ that are such that $v > c$, if buyers with willingness to trade v and sellers with costs c behave as*

[5] Note that convergence of beliefs implies that the buyers' and sellers' beliefs about the distribution of the price overlap. In Rustichini et al. (2004) the centralized market price is volatile because the number of traders is finite, and convergence of beliefs is obtained as a result in their paper. The same is true in the models of Cripps and Swinkels and Reny and Perry. Cripps and Swinkels explicitly allow for aggregate uncertainty and require only "a little independence." Reny and Perry assume that values are interdependent, but that types are independent. In both of these models convergence of beliefs follows from independence and the fact that the markets are large. Volatility follows from the fact that the number of traders is finite, and in the case of Cripps and Swinkels, also from the fact that participation in the market is stochastic.

[6] Notice that a buyer who bids his true valuation v would be rationed only in the event that the market price happens to equal to v and that in this case rationing does not affect the buyer's payoff. The same is true for sellers below.

price-takers in the centralized market (that is, the buyers bid $b(v) = v$ and the sellers ask for $a(c) = c$), then for every $\varepsilon > 0$ there exists a large enough number of buyers and sellers such that if the number of buyers and sellers in the centralized market is larger, then

$$B_t^M(v) + S_t^M(c) \geq v - c - \varepsilon \tag{1}$$

regardless of whether other buyers and sellers behave as price-takers.

Proof. Observe that

$$\begin{aligned} S_t^M(c) &= \int_c^1 (p - c)\, dQ_t(c)(p) \\ &= \int_0^1 (p - c)\, dQ_t(c)(p) + \int_0^c (c - p)\, dQ_t(c)(p) \\ &\geq E\left[p_t^M \,|c\right] - c \end{aligned}$$

and

$$\begin{aligned} B_t^M(v) &= \int_0^v (v - p)\, dQ_t(v)(p) \\ &= \int_0^1 (v - p)\, dQ_t(v)(p) + \int_v^1 (p - v)\, dQ_t(v)(p) \\ &\geq v - E\left[p_t^M \,|v\right] \end{aligned}$$

Together, the previous two inequalities imply that for every $c, v \in \mathcal{G}$, if a seller c and a buyer v behave as price-takers, then

$$B_t^M(v) + S_t^M(c) \geq v - c - \left(E\left[p_t^M \,|v\right] - E\left[p_t^M \,|c\right]\right).$$

Convergence of beliefs implies that for every $\varepsilon > 0$, there exists a number of traders such that for any larger expected number of buyers and sellers in the centralized market (1) follows. ■

The fact that buyers and sellers in the centralized market are not obliged to behave as price-takers makes the centralized market even more attractive to them than what is suggested by inequality (1). Together with our assumption that $\delta < 1$, this implies that buyers and sellers are always strictly better off opting for the centralized market than refraining from trade because instead of waiting for a future period in which the price is expected to be more favorable, a trader can bid or ask for this price in the centralized market already in the current period.

3.2. Direct Negotiation

As in the case of centralized markets, we also adopt a reduced form approach to model the process of direct negotiations among the traders. For every willingness to pay v and cost c, let $\mu_t^B(v)$ and $\mu_t^S(c)$ denote the number of buyers with willingness to pay v and the number of sellers with cost c, respectively, who opt for trading through direct negotiations in period t. We assume that traders who opted for direct negotiations are matched into pairs of one buyer and one seller according to the probability function $f_t(v,c)$.[7] Conditional on being matched, a buyer and seller with types v and c, respectively, trade with each other with probability $x_t(v,c)$, at an expected price $p_t^N(v,c)$.

We assume that:

1. For every $t \geq 1$, if $f_t(v,c) > 0$ or $x_t(v,c) > 0$ for some $c, v \in \mathcal{G}$ that are such that $v - c \geq \dfrac{1}{K}$ then also

   $$f_t\left(v, c+\frac{1}{K}\right), f_t\left(v-\frac{1}{K}, c\right) > 0 \text{ and } x_t\left(v, c+\frac{1}{K}\right), x_t\left(v-\frac{1}{K}, c\right) > 0,$$

   respectively.

2. For every $t \geq 1$, and buyer's and seller's types $c, v \in \mathcal{G}$,

   $$c \leq p_t^N(v,c) \leq v.$$

3. The price function $p_t^N(v,c)$ is nondecreasing in v and c, and strictly increasing in either v or c.

The first assumption captures the idea that if a buyer v is matched and trades with a seller c with a positive probability, then it cannot avoid also being matched and trading with a seller whose cost is a little higher if it is

[7] That is a buyer with willingness to pay v is matched with a seller with cost c in period t with probability

$$f_t(v,c)\min\left\{\frac{\mu_t^S(c)}{\mu_t^B(v)}, 1\right\}$$

and a seller with cost c is matched with a buyer with willingness to pay v with probability

$$f_t(v,c)\min\left\{\frac{\mu_t^B(v)}{\mu_t^S(c)}, 1\right\}.$$

present (provided, of course, that such a transaction is not inefficient), and vice-versa. The second assumption reflects the fact that trade is voluntary. And, the third assumption captures the intuition that exactly because of their "eagerness," eager buyer types (i.e., buyers with a high willingness to pay) are likely to pay relatively higher prices and eager seller types (seller with low costs) are likely to accept relatively lower prices. This assumption is satisfied in many models of bargaining including Nash's (1950) model of axiomatic bargaining, Rubinstein's (1982) model of alternating offers bargaining, and Myerson and Satterthwaite's (1983) optimal mechanism for bilateral bargaining under asymmetric information.

The assumption that $p_t^N(v,c)$ is strictly increasing in at least one of its arguments is the main assumption that distinguishes our model of direct negotiations from our model of a centralized market, where the effect that individual traders have on the price at which they transact is assumed to vanish as the centralized market becomes large, and consequently, eager types are not disadvantaged because of their eagerness.

Given $\mu_t^B(v)$, $\mu_t^S(c)$, $f_t(\cdot,\cdot)$, $x_t(\cdot,\cdot)$, and $p_t^N(\cdot,\cdot)$, denote the expected payoffs conditional on trade from opting for direct negotiations in period t of the buyers and sellers by $B_t^{N|trade}(v)$ and $S_t^{N|trade}(c)$, respectively.[8] Our assumptions about $f_t(\cdot,\cdot)$, $x_t(\cdot,\cdot)$, and $p_t^N(\cdot,\cdot)$, imply that the two functions $B_t^{N|trade}(v)$ and $S_t^{N|trade}(c)$ satisfy the following property, which is employed repeatedly in the proof below.

Lemma 2. *For any cost and willingness to pay* $0 \leq c^* < v^* \leq 1$, *if all the sellers with costs* c^* *who opt for direct negotiations in some period* t *trade with buyers with willingness to pay* $v \leq v^*$, *and all the buyers with willingness to*

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[8] That is, $B_t^{N|trade}$ is given by

$$B_t^{N|trade}(v) = \frac{\sum_{c\in\mathcal{G}} \left(v - p_t^N(v,c)\right) x_t(v,c) f_t(v,c) \min\left\{\frac{\mu_t^S(c)}{\mu_t^B(v)}, 1\right\}}{\sum_{c\in\mathcal{G}} x_t(v,c) f_t(v,c) \min\left\{\frac{\mu_t^S(c)}{\mu_t^B(v)}, 1\right\}}$$

if buyer v trades with a positive probability ($\sum_{c\in\mathcal{G}} x_t(v,c) f_t(v,c) \min\left\{\frac{\mu_t^S(c)}{\mu_t^B(v)}, 1\right\} > 0$) and is assumed to be equal to zero otherwise, and $S_t^{N|trade}$ is given by

$$S_t^{N|trade}(c) = \frac{\sum_{c\in\mathcal{G}} \left(p_t^N(v,c) - c\right) x_t(v,c) f_t(v,c) \min\left\{\frac{\mu_t^B(v)}{\mu_t^S(c)}, 1\right\}}{\sum_{c\in\mathcal{G}} x_t(v,c) f_t(v,c) \min\left\{\frac{\mu_t^B(v)}{\mu_t^S(c)}, 1\right\}}$$

if seller c trades with a positive probability and is assumed to equal to zero otherwise.

pay v^* who opt for direct negotiations at t trade with sellers with costs $c \geq c^*$, then

$$B_t^{N|trade}(v^*) + S_t^{N|trade}(c^*) \leq v^* - c^* - \Delta. \tag{2}$$

for some $\Delta > 0$.

Proof. Fix some $0 \leq c^* < v^* \leq 1$. Our assumptions about $f_t(\cdot,\cdot)$ and $x_t(\cdot,\cdot)$ imply that if a buyer v^* trades with a positive probability, then it trades with a positive probability with a seller of type $c \geq c^* + \frac{1}{K}$, and if a seller c^* trades with a positive probability, then it trades with a positive probability with a buyer of type $v \leq v^* - \frac{1}{K}$.

The monotonicity of the price function $p_t^N(\cdot,\cdot)$ then implies that

$$B_t^{N|trade}(v^*) \leq v^* - p_t^N(v^*, c^*)$$

because conditional on trade, every buyer v^* pays a price that is larger than or equal to $p_t^N(v^*, c^*)$. A similar argument implies that

$$S_t^{N|trade}(c^*) \leq p_t^N(v^*, c^*) - c^*.$$

Moreover, the fact that $p_t^N(\cdot,\cdot)$ is strictly increasing in either v or c implies that at least one of the previous two inequalities is strict. Adding these two inequalities together, it follows that

$$B_t^{N|trade}(v^*) + S_t^{N|trade}(c^*) < v^* - c^*.$$

Finally, the value of Δ is given by

$$\Delta = \min_{c^*, v^* \in \mathcal{G}, c^* < v^*} \left\{ v^* - c^* - B_t^{N|trade}(v^*) - S_t^{N|trade}(c^*) \right\}$$

where the minimum is taken over all the pairs $0 \leq c^* < v^* \leq 1$. Because the minimum is obtained on some pair c^*, v^*, it follows that $\Delta > 0$. ■

Many different forms of direct negotiations satisfy inequality (2) above. We describe three examples of such direct negotiation procedures below.

Example 1 (Direct Negotiations). Fix a sequence of real numbers $\{\alpha_t\}_{t\in\{1,2\ldots\}}$ such that for every t, $\alpha_t \in (0,1)$. Suppose that in every period t the procedure of direct negotiations between the buyers and sellers assumes the form of random matching into pairs of one buyer and one seller, followed by split-the-surplus bilateral bargaining where buyers capture a fraction $\alpha_t \in (0,1)$ of the available surplus while the sellers get the rest. When a buyer with willingness to pay v and a seller with cost c are matched in period t, they transact at the price $\alpha_t c + (1 - \alpha_t) v$ if $v \geq c$ and refrain from trade otherwise.

The price function continues to satisfy the restrictions specified above also if each period is divided into a number of sub-periods, and traders who were matched with partners with whom they could not profitably trade in any sub-period, are randomly re-matched again in the next sub-period.

Example 2 (A Dealers' Market). This example is based on Spulber (1996). A dealer market consists of a large number of heterogenous buyers, sellers, and middlemen. Traders and middlemen discount future profits at a rate $\delta < 1$, and in every period, each trader exits the market with an exogenously specified probability $\lambda > 0$. The initial distribution of buyers' and sellers' types is the uniform distribution on the grid $\mathcal{G}$, and whenever a buyer or seller trades or exits the market, it is replaced by another buyer or seller whose type is drawn from the uniform distribution on $\mathcal{G}$. The only way for buyers and sellers to trade is through middlemen who quote bid and ask prices. Thus, a match in this market is between a trader and a middleman. The middlemen are infinitely lived and each set a pair of stationary bid and ask prices to maximize their expected discounted profits. The middlemen are each characterized by their transaction costs that are uniformly distributed over the interval $[0, 1]$. A middleman with transaction cost (type) $k \in [0, 1]$ sets bid and ask prices $a(k)$ and $b(k)$, respectively. Buyers and sellers engage in sequential search. Each period, a searcher obtains a single price quote from one, randomly drawn, middleman. It can be shown that this market has a unique stationary equilibrium. In this equilibrium, bid and ask prices are uniformly distributed over some interval. The fact that in this equilibrium buyers with a higher willingness to pay and sellers with lower costs are willing to buy from middlemen who quote higher and lower ask and bid prices, respectively, implies that inequality (2) is satisfied.

Example 3 (Parallel Auctions). In every period t, each seller sells its object through an auction, and buyers choose in which seller's auction to participate. Suppose that sellers' may each specify a reserve price, and buyers choose randomly among sellers who specified the same reserve price. The fact that in equilibrium, in all "standard auctions" (English, Dutch, first-price, second-price, all-pay) buyers with high willingness to pay pay more in expectation, and that sellers' optimal reservation prices are nondecreasing in their costs, implies that inequality (2) is satisfied.

A large literature has analyzed the conditions under which "frictionless" decentralized bargaining may give rise to the Walrasian outcome, in which all buyers with willingness to pay above the Walrasian price and all sellers with costs below the Walrasian price trade at the Walrasian price (for a survey of this literature, see Osborne and Rubinstein, 1990). This literature has shown that if the following conditions are all satisfied:

1. the number of buyers and sellers is very large,
2. the traders are infinitely patient,
3. the traders are anonymous, and
4. there is no aggregate uncertainty,

then the Walrasian outcome may prevail (see, e.g., Gale, 1986, 1987). However, if decentralized bargaining is not "frictionless," or more specifically, if traders are *not* infinitely patient (that is, traders' discount factor δ is strictly less than 1) or if there is aggregate uncertainty, then the conclusion of this literature is that the Walrasian outcome is impossible (Gale, 2000).

The Walrasian outcome is incompatible with our assumptions about trade under direct negotiations because it is incompatible with the conclusion of Lemma 2. Thus, a direct negotiation procedure in which in every period trade is taking place at the expected price in the centralized market is inconsistent with our assumptions.

4. Equilibrium Analysis

In this section, we show that in any given period, only two types of outcome are consistent with Nash equilibrium: either all those traders who trade do so through the centralized market, or possibly some traders, but not many, trade through the centralized market, while all the others trade through direct negotiation. We show that while the former type of equilibrium outcome arises in a strong Nash equilibrium (Aumann, 1959), which implies that it is immune to an improving deviation by any coalition of traders, the latter equilibrium outcome arises in an equilibrium that is not coalition proof (Bernheim et al., 1987). Moreover, the latter equilibrium can be destabilized by a self-enforcing deviation of a very small coalition of traders.

The idea of the proof is to show that in any period in which the centralized market attracts a sufficiently large number of traders, trade through direct negotiations "unravels" as traders who have relatively more eager types switch to the centralized market. If, however, the centralized market fails to attract a sufficiently large number of traders in any given period, then some traders, but not too many, might trade through the centralized market while others might trade through direct negotiations.

Because a trader who fails to trade at t may trade at a future period, it does not necessarily follow that traders would prefer to trade through the exchange mechanism that provides the higher expected payoff at t.[9] However, as the next lemma shows, traders would prefer the centralized market if their (unconditional) expected payoff there when they behave competitively is larger than their expected payoff conditional on trade in direct negotiations.

Lemma 3. *For every period $t \in \{1, 2, ...\}$ and for every buyer with willingness to pay v who opts for trading in period t: if $B_t^M(v) > B_t^{N|trade}(v)$, then the buyer strictly prefers to trade through the centralized market than to trade through direct negotiations at t. Similarly, for a seller with cost c, if $S_t^M(c) > S_t^{N|trade}(c)$, then the seller strictly prefers to trade through the centralized market than to engage in direct negotiations at t.*

Proof. We prove the lemma for buyers. The proof for sellers is similar. Suppose that

$$B_t^M(v) > B_t^{N|trade}(v).$$

[9] Suppose for example that in every period, in the centralized market a trader trades with probability $\frac{2}{3}$ and obtains an expected payoff conditional on trade of $\frac{1}{2}$, and in direct negotiations the trader trades with probability $\frac{1}{9}$ and obtains an expected payoff conditional on trade of $\frac{9}{10}$. Although in any given period the expected payoff from trading in the centralized market is higher $\left(\frac{1}{3} > \frac{1}{10}\right)$, a patient trader would maximize his payoff by repeatedly trying to trade though direct negotiations.

We introduce the following notation: let

$$\begin{aligned}
B_t^{M|trade}(v) &= \text{the expected payoff conditional on trade of a buyer whose willingness to pay is } v \text{ from participating in the centralized market in period } t; \\
B_t^N(v) &= \text{the expected payoff of a buyer with willingness to pay } v \text{ from engaging in direct negotiations in period } t; \\
P_t^M(v) &= \text{the probability that a buyer with a willingness to pay } v \text{ trades in the centralized market in period } t; \\
P_t^N(v) &= \text{the probability that a buyer with a willingness to pay } v \text{ trades through direct negotiations in period } t; \\
B_{t\to\infty}(v) &= \text{the expected discounted payoff of a buyer with willingness to pay } v \text{ in period } t \text{ who chooses optimally whether and where to trade.}
\end{aligned}$$

For any period t, a buyer with willingness to pay v strictly prefers to trade through the centralized market than to engage in direct negotiations if and only if his expected discounted payoff from doing so is higher, or

$$\begin{aligned}
&P_t^M(v)B_t^{M|trade}(v) + \left(1 - P_t^M(v)\right)\delta B_{t+1\to\infty}(v) \\
&\qquad > P_t^N(v)B_t^{N|trade}(v) + \left(1 - P_t^N(v)\right)\delta B_{t+1\to\infty}(v). \quad (3)
\end{aligned}$$

Because $B_t^M(v) = P_t^M(v)B_t^{M|trade}(v) \geq 0$, and $0 \leq P_t^M(v) \leq 1$,

$$B_t^{M|trade}(v) \geq B_t^M(v).$$

Similarly,

$$B_t^{N|trade}(v) = \frac{B_t^N(v)}{P_t^N(v)} \geq B_t^N(v).$$

The fact that $B_t^M(v) > B_t^{N|trade}(v)$ implies that both

$$B_t^{M|trade}(v) > B_t^{N|trade}(v)$$

and

$$B_t^M(v) > B_t^N(v).$$

Unless $B_t^{M|trade}(v) \geq \delta B_{t+1\to\infty}(v)$ the buyer is better off refraining from trade in period t. Because the Lemma only applies to buyers who opt for trading at t, we may also assume that $B_t^{M|trade}(v) > \delta B_{t+1\to\infty}(v)$ (note that if $B_t^{M|trade}(v) = \delta B_{t+1\to\infty}(v)$, then $B_t^{M|trade}(v) > B_t^{N|trade}(v)$ implies that the buyer is strictly better off trading through the centralized market at t).

Observe that inequality (3) can be rewritten as

$$P_t^M(v) > P_t^N(v)\frac{B_t^{N|trade}(v) - \delta B_{t+1\to\infty}(v)}{B_t^{M|trade}(v) - \delta B_{t+1\to\infty}(v)}, \tag{4}$$

and because $P_t^M(v)B_t^{M|trade}(v) = B_t^M(v)$ and $P_t^N(v)B_t^{N|trade}(v) = B_t^N(v)$, also as

$$P_t^M(v) < P_t^N(v) + \frac{B_t^M(v) - B_t^N(v)}{\delta B_{t+1\to\infty}(v)}. \tag{5}$$

If $P_t^M(v) \geq P_t^N(v)$, then $B_t^{M|trade}(v) > B_t^{N|trade}(v)$ implies (4) and so (3). And if $P_t^M(v) \leq P_t^N(v)$, then $B_t^M(v) > B_t^N(v)$ implies (5) and so again (3).

Finally, the fact that traders are not obliged to behave competitively or as price-takers in the centralized market makes it even more attractive for them. ■

Every Nash equilibrium sequence of distributions of buyers' and sellers' types in the centralized market and direct negotiations, respectively, induces a sequence of functions $\left\{B_t^M(\cdot)\right\}_{t\geq 1}$, $\left\{S_t^M(\cdot)\right\}_{t\geq 1}$, $\left\{B_t^{N|trade}(\cdot)\right\}_{t\geq 1}$, and $\left\{S_t^{N|trade}(\cdot)\right\}_{t\geq 1}$ that denote the expected payoffs and the conditional expected payoffs to buyers and sellers if they were to trade competitively through the centralized market or trade through direct negotiations, respectively. Given these sequences of functions define two sequences of thresholds $\{v_t^*\}_{t\geq 1}$ and $\{c_t^*\}_{t\geq 1}$ as follows: for every $t \geq 1$,

$$v_t^* \equiv \min\left\{v \in \mathcal{G} : B_t^M(v') > B_t^{N|trade}(v') \text{ for every } v' > v\right\},$$

and

$$c_t^* \equiv \max\left\{c \in \mathcal{G} : S_t^M(c') > S_t^{N|trade}(c') \text{ for every } c' < c\right\}.$$

Lemma 3 implies that v_t^* is (the lowest possible) threshold willingness to pay above which every buyer's type who opts for trading at t opts for the centralized market, and c_t^* is (the highest possible) threshold cost below which every seller's type who opts for trading at t opts for the centralized market.

Observe that because the threshold v_t^* may possibly be equal to one and the threshold c_t^* may possibly be equal to zero both v_t^* and c_t^* are well defined for every $t \geq 1$. We show that for every $t \in \{1, 2, ...\}$ in which the centralized market attracts a sufficiently large number of traders, $0 \leq v_t^* \leq c_t^* \leq 1$.

Lemma 4. *In every Nash equilibrium, in every period $t \in \{1, 2, ...\}$ in which the centralized market attracts a sufficiently large number of traders so that $B_t^M(v) + S_t^M(c) \geq v - c - \frac{\Delta}{2}$ for every $v, c \in \mathcal{G}$, $0 \leq v_t^* < c_t^* \leq 1$.*

Proof. Fix some Nash equilibrium. Fix some $t \in \{1, 2, ...\}$ in which the number of traders who opt for the centralized market is sufficiently large so that $B_t^M(v) + S_t^M(c) \geq v - c - \frac{\Delta}{2}$ for every $v, c \in \mathcal{G}$. Suppose that $c_t^* < v_t^*$. We show that this implies a contradiction. The definitions of c_t^* and v_t^* imply that sellers with costs c_t^* who opt for direct negotiations at t trade with buyers with willingness to pay $v \leq v_t^*$, and buyers with willingness to pay v_t^* who opt for direct negotiations at t trade with sellers with costs $c \geq c_t^*$. It therefore follows that

$$\begin{aligned} v_t^* - c_t^* &\leq B_t^M(v_t^*) + S_t^M(c_t^*) + \frac{\Delta}{2} \\ &\leq B_t^{N|trade}(v_t^*) + S_t^{N|trade}(c_t^*) + \frac{\Delta}{2} \\ &< v_t^* - c_t^*, \end{aligned}$$

where the first inequality follows from the condition of the lemma, the second inequality follows from the definitions of c_t^* and v_t^*, and the third inequality follows from Lemma 1. A contradiction.

The previous argument showed that $v_t^* \leq c_t^*$. To prove that $v_t^* < c_t^*$, we show that it cannot be that $v_t^* = c_t^*$. Suppose that $v_t^* = c_t^*$. The definitions of v_t^* and c_t^* imply that $B_t^M(v_t^*) \leq B_t^{N|trade}(v_t^*)$ and $S_t^M(c_t^*) \leq S_t^{N|trade}(c_t^*)$. The assumption that $c_t^* \leq p_t^N(v_t^*, c_t^*) \leq v_t^*$ implies that $B_t^{N|trade}(v_t^*) = S_t^{N|trade}(c_t^*) = 0$. But $B_t^M(v_t^*) = \int_0^{v_t^*} (v_t^* - p)\, dQ_t(v_t^*)(p) > 0$ if the price in the centralized market, as perceived by a buyer with willingness to pay v_t^*, decreases below v_t^* with a positive probability, and $S_t^M(c_t^*) = \int_{c_t^*}^1 (p - c_t^*)\, dQ_t(c_t^*)(p) > 0$ if the price in the centralized market, as perceived by a seller with cost c_t^*, increases above c_t^* with a positive probability. Convergence of beliefs plus the fact that the centralized market price is volatile implies that the centralized market price as perceived by either a buyer with willingness to pay v_t^* or a seller with cost c_t^* either decreases below v_t^* with a positive probability or increases above $c_t^* = v_t^*$ with a positive probability. A contradiction. ∎

We summarize our results in the following proposition.

Proposition. *In every Nash equilibrium, in every period in which the centralized market attracts a sufficiently large number of traders so that* $B_t^M(v)+S_t^M(c) \geq v-c-\frac{\Delta}{2}$ *for every* $v, c \in [0,1]$, *all those buyers and sellers who trade, trade through the centralized market. No trade occurs through direct negotiations.*

Proof. Lemma 4 implies that in every period in which the centralized market attracts a sufficiently large number of traders $v_t^* < c_t^*$. The definition of v_t^* and c_t^* implies that in this case all the buyers and sellers who opt for direct negotiations have willingness to pay smaller than or equal to v_t^* and costs larger than or equal to c_t^*, respectively. It therefore follows that no opportunities for mutually beneficial trade exist outside the centralized marketplace at t. ■

The intuition for this result is the following. The surplus that is generated by a buyer of type v and a seller of type c is $\max\{v-c, 0\}$. Inequality (1) may thus be interpreted as implying that centralized markets allow eager traders to keep almost the entire surplus they generate. In contrast, in direct negotiations, as Lemma 1 shows, eager types of traders are forced to share the surplus they generate with others. This difference between the centralized market and direct negotiation, which causes relatively eager types of traders to prefer the centralized market and the unraveling of direct negotiations, is due to the stronger impact that a higher willingness to pay and cost have on transaction prices in direct negotiations compared to a large centralized market.

Specifically, fix for some period t distributions of buyers' and sellers' types in the centralized market and direct negotiations, respectively. Suppose that the induced functions $B_t^M(\cdot)$, $S_t^M(\cdot)$, $B_t^{N|trade}(\cdot)$, and $S_t^{N|trade}(\cdot)$, give rise to the thresholds $v_t^* < 1$ and $c_t^* > 0$, respectively. These functions and thresholds are depicted in the figure below.

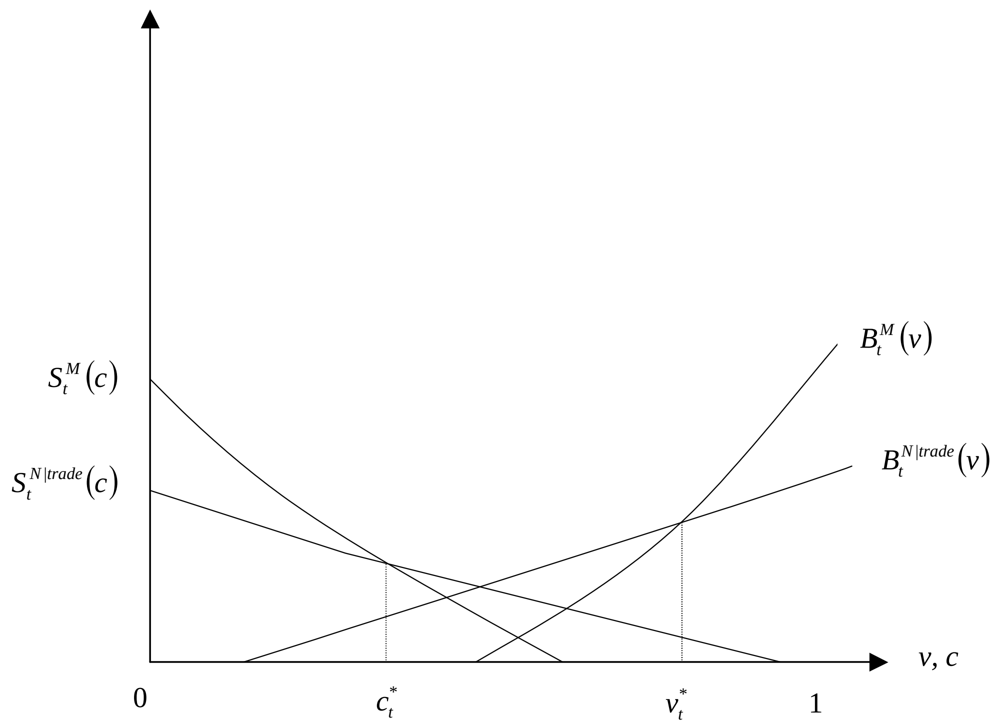

Figure 1: B_t^M, S_t^M, $B_t^{N|trade}$, $S_t^{N|trade}$, v_t^*, and c_t^*

As can be seen in the figure, buyers with willingness to pay above v_t^* and sellers with costs below c_t^*, who according to the original distributions opted for direct negotiations would be better off switching to the centralized market. The functions $B_t^M(\cdot)$, $S_t^M(\cdot)$, $B_t^{N|trade}(\cdot)$, and $S_t^{N|trade}(\cdot)$, and the thresholds v_t^* and c_t^* should therefore be recomputed given this switch. The expected price in the centralized market and so also $B_t^M(\cdot)$ and $S_t^M(\cdot)$ may not be much affected by such a switch. But, because it is the buyers with relatively high willingness to pay and sellers with relatively low cost (the eager types) who switch to the centralized market, the switch would have a more dramatic effect on the expected payoffs in direct negotiations, $B_t^{N|trade}(\cdot)$ and $S_t^{N|trade}(\cdot)$, because after the switch the distributions of buyers' and sellers' types in direct negotiations would be more concentrated on buyer types with low willingness to pay and seller types with high cost. Consequently, both the recomputed functions $B_t^{N|trade}(\cdot)$ and $S_t^{N|trade}(\cdot)$, and the recomputed threshold v_t^* would be lower than they were before, and the recomputed threshold c_t^* would be higher than it was before. The process of unraveling would then continue as buyers with willingness to pay above the recomputed v_t^* and sellers with cost below the recomputed c_t^* would want to switch to the centralized market and so on, until the recomputed v_t^* and c_t^* would be such that $0 \leq v_t^* < c_t^* \leq 1$. At this point all the "serious" traders, that is all the traders' types that are in fact likely to trade in the centralized market if they were to opt to trade there, would trade through the centralized market, so that no opportunities for mutually beneficial trade would remain under direct negotiations.

The proposition implies that there exist two types of equilibria: one in which in all those who trade opt for the centralized market in every period, and one in which in some periods, the number of traders who opt for the centralized market is small, and consequently either most of those who trade do so through direct negotiations, or not much trade is taking place. We show that the equilibrium where all those who trade opt for the centralized market in every period is more "stable" and hence more plausible than the equilibrium where all those who trade do so through direct negotiations in the following sense: while the former equilibrium is a strong Nash equilibrium (Aumann, 1959), the latter is not even a coalition proof Nash equilibrium (Bernheim et al., 1987), which is a weaker concept.

Aumann (1959) defines a strong Nash equilibrium as a Nash equilibrium that is immune to an improving deviation by any coalition of players. Formally, a profile of strategies σ is a strong Nash equilibrium if there does not exist a coalition of players $G \subseteq N$ and a profile of strategies that is played by the members of the coalition G, denoted σ'_G, that is such that the payoff to each member of the coalition G under the profile of strategies $\left(\sigma'_G, \sigma_{N \backslash G}\right)$ is larger than or equal to the payoff to the member under the original profile σ, and for at least one member of G, the payoff under $\left(\sigma'_G, \sigma_{N \backslash G}\right)$ is strictly larger than under original profile σ.

Bernheim et al. (1987) define a coalition proof Nash equilibrium (CPNE) as a Nash equilibrium that is immune to improving deviations that are self-enforcing. A deviation is self-enforcing if there is no further self-enforcing and improving deviation available to a proper subcoalition of the deviating players. Formally, a profile of strategies σ is a coalition proof Nash equilibrium if there does not exist a coalition of players $G \subseteq N$ and a profile of strategies that is played by the members of the coalition G, denoted σ'_G, that is such that the payoff to each member of the coalition G under the profile of strategies $\left(\sigma'_G, \sigma_{N \backslash G}\right)$ is larger than or equal to the payoff to the member under the original profile σ, and for at least one member of G, the payoff under $\left(\sigma'_G, \sigma_{N \backslash G}\right)$ is strictly larger than under original profile σ, and such that the deviation of the coalition G is self enforcing. A deviation of a coalition G is self-enforcing if there is no further self-enforcing and improving deviation available to a proper subcoalition G' of the deviating coalition G.[10]

A strong Nash equilibrium is therefore a CPNE, but a CPNE need not be a strong Nash equilibrium.

Thus, if we believe that the traders can communicate and coordinate their strategies, then a strong Nash equilibrium is a much more plausible prediction of the way they will play than a Nash equilibrium that is not coalition-proof.

We show that the Nash equilibrium where in every period every trader opts for the centralized market, and where consequently convergence of beliefs is achieved, is a strong Nash equilibrium. In contrast, any Nash equilibrium in which in some period some traders trade through direct negotiations is not a coalition proof Nash equilibrium.

Lemma 5. *The Nash equilibrium where in every period every trader opts for the centralized market, and where consequently price convergence in the centralized market is achieved in every period, is a strong Nash equilibrium.*

Proof. Consider the profile of traders' strategies that is such that in every period every trader opts for the centralized market. Since no trader ever opts for direct negotiation and traders are free to ask for or bid any price in the centralized market, this profile of strategies is indeed a Nash equilibrium. Fix a period t. We show that any deviation by a coalition of players hurts at least some members of the deviating coalition.

We first show that no coalition can benefit from a deviation in which its members opt for trading through direct negotiations instead of the centralized market in period t. Denote the maximum willingness to pay of a deviating buyer and the minimum cost of a deviating seller by $v_{\max}$ and $c_{\min}$, respectively. By definition, no buyer with willingness to pay above $v_{\max}$ and no seller with cost below $c_{\min}$ opt for direct negotiations. By Lemma 1

$$B_t^{N|trade}(v_{\max}) + S_t^{N|trade}(c_{\min}) \leq v_{\max} - c_{\min} - \Delta \tag{6}$$

while by (1)

$$B_t^M(v_{\max}) + S_t^M(c_{\min}) \geq v_{\max} - c_{\min} \tag{7}$$

(since without a deviation all the traders opt for the centralized market, and so convergence of beliefs implies that $\varepsilon = 0$). It follows that either $B_t^M(v_{\max}) > B_t^{N|trade}(v_{\max})$ or $S_t^M(c_{\min}) > S_t^{N|trade}(c_{\min})$. That is, the deviation cannot be improving for either $v_{\max}$ or $c_{\min}$.

[10] We allow membership in the deviating coalition G to be stochastic, which implies that we consider a slightly stronger notions of strong Nash and coalition proof Nash equilibrium.

The same argument implies that a coalition that deviates by postponing its trade to a later period $t' > t$, would be better off opting for the centralized market rather than direct negotiation in period t'.

Because we consider a Nash equilibrium, a single trader cannot benefit by not trading in period t. The definition of the rules of trade in the centralized market a deviating coalition of buyers or a deviating coalition of sellers that declines to trade in period t and instead opts for trading in the centralized market in period $t' > t$ moves the price against itself relative to the price that would be faced by a single buyer or seller. Thus, the fact that a single buyer or a single seller cannot benefit from postponing its trade implies that a coalition of buyers or a coalition of sellers cannot benefit from postponing its trade either. Finally, a coalition of buyers *and* sellers cannot benefit from postponing its trade to a later period t' in which it opts for the centralized market because it is impossible that both deviating buyers and deviating sellers would all be better off. Buyers can be better off only if the expected price at t' is lower than the expected price at t, and sellers can be better off only if the expected price at t' is higher than the expected price at t. This is impossible if there is price convergence at both t and t'. ∎

Lemma 6. *A Nash equilibrium in which in some period some traders trade through direct negotiations is not a coalition proof Nash equilibrium.*

Proof. Consider a Nash equilibrium where in some period t some traders trade through direct negotiations. Consider a coalition of traders that (i) includes only buyers with the highest willingness to pay and sellers with the lowest cost, (ii) is large enough to ensure convergence of beliefs in the centralized market if it were to opt to trade there, and (iii) is chosen such that the expected price in the centralized market if the coalition opts to trade there, denoted p^M, is equal to $p_t^N(1,0)$. Since if the traders in the coalition all behave competitively in the centralized market and the number of buyers in the coalition is larger than the number of sellers then the price in the centralized market is one, and if the traders in the coalition all behave competitively and the number of sellers in the coalition is larger than the number of buyers then the price in the centralized market is zero, it is possible to select such a coalition albeit with a possibly random number of buyers and sellers.

We show that the members of the coalition are all made better off by jointly deviating to the centralized market. The expected payoff to a seller who is a member of the coalition and who behaves competitively in the centralized market is

$$\begin{aligned} S_t^M(0) &= \int_0^1 p dQ_t(0)(p) \\ &= p^M \end{aligned}$$

while monotonicity of the price function in direct negotiations plus the fact that a seller with cost 0 cannot avoid being matched and trading with a buyer whose willingness to pay is less than 1 implies that

$$\begin{aligned} S_t^{N|trade}(0) &\leq p_t^N(1,0) \\ &= p^M. \end{aligned}$$

Similarly, the expected payoff to a buyer who is a member of the coalition and who behaves competitively in the centralized market is

$$\begin{aligned} B_t^M(1) &= \int_0^1 (1-p)\, dQ_t(1)(p) \\ &= 1 - p^M \end{aligned}$$

while monotonicity of the price function in direct negotiations plus the fact that a buyer with willingness to pay 1 cannot avoid being matched and trading with a seller whose cost is strictly more than 0 implies that

$$\begin{aligned} B_t^{N|trade}(1) &\leq 1 - p_t^N(1,0) \\ &= 1 - p^M. \end{aligned}$$

The assumption that $p_t^N(\cdot,\cdot)$ is strictly increasing in at least one of its arguments implies that at least one of the previous two inequalities is strict. ■

Finally, we note that not only is it the case that the Nash equilibrium where some trade occurs through direct negotiations is not a coalition proof Nash equilibrium, but that inspection of the proof of Lemma 6 reveals that the size of the coalition that destabilizes it is very small, proportionally to the total number of traders, because it consists only of the traders with the most extreme types.

5. Concluding Remarks

The argument presented here is that when faced with the choice, buyers and sellers will opt for trading through a centralized market over engaging in (some form of) direct negotiations. Nevertheless, some transactions, even in homogenous goods, are still conducted through direct negotiations. A number of possible explanations may be given for this. We discuss these explanations in the context of the model described in this paper.

First, it may be that the traded good is not really homogenous. In such a case, problems associated with quality and credibility may arise, and traders may prefer the relative security of establishing long term trading relationships with a small number of trustworthy trading partners, where the prospect of engaging in future trade serves as a disciplinary device against opportunistic behavior, to trading in an anonymous centralized market where relatively little protection against opportunistic behavior is provided (Kranton, 1996).[11]

Second, participation in a centralized market may entail some costs that we have not taken into account here (transportation costs and the fact that some markets convene only infrequently are possible examples). However, if these costs are similar to those incurred under direct negotiations, our main result should still hold.

Third, ensuring the constant operation of the centralized market, which is necessary for our main result, is a public good, or more precisely, a public service. A centralized market may not dominate other forms of exchange if no one is willing to assume the responsibility for the orderly provision of this public service. As shown by Rust and Hall (2003), if the centralized market is organized by a market maker who charges a positive bid-ask spread, then the unraveling of direct negotiations will not be complete.

Fourth, we have assumed in our analysis that traders are risk neutral. Another reason to prefer direct negotiations over a centralized market is that the former may allow *risk averse* traders to reduce their exposure to the centralized market's volatility by directly negotiating to trade at the *expected* centralized market price. However, because more risk averse or pessimistic traders should also be willing to pay to reduce their exposure to risk, an argument similar to the one presented in this paper implies that a centralized *futures* market that insures against this volatility will again dominate decentralized private mutual insurance agreements.

[11] However, even in Kranton's (1996) model, all trade will eventually be conducted through the centralized market if its initial size is sufficiently large.

Finally, even when, say, because of the presence of transaction costs in the centralized market, the unraveling of trade outside the centralized market does not go all the way towards eliminating trade through direct negotiations, our model still provides an insight about the relative willingness of different types of traders to trade through different forms of exchange. Centralized markets are characterized by the fact that eager traders keep almost the entire additional marginal surplus generated by their type. In contrast, in many models of negotiations (Nash, Rubinstein, Myerson-Satterthwaite) traders are forced to share this surplus with others. This causes eager traders to prefer centralized markets, which would cause direct negotiations to unravel, wholly or partially. Recent experimental work by Kugler et al. (2006) confirms this prediction.

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