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The asymptotic strategyproofness of scoring and Condorcet consistent rules

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Abstract. We calculate the proportion of preference profiles where "small" coalitions of agents may successfully manipulate any given scoring rule and show that it decreases to zero at a rate proportional to $\frac{1}{\sqrt{n}}$ with the number of agents. If agents have to incur a small cost in order to decide how to manipulate the voting rule, our results imply that scoring rules are robust to such manipulation in large groups of agents. We present examples of asymptotically strategyproof and non strategyproof Condorcet consistent rules.

1 Introduction

We consider a social decision problem where a group consisting of n agents is required to choose one out of K different alternatives. We calculate the proportion of preference profiles where agents, or "small" coalitions of agents, may successfully manipulate scoring rules and Condorcet consistent rules. We show that under any scoring rule, the proportion of manipulable preference profiles decreases to zero at a rate proportional to $\frac{1}{\sqrt{n}}$. Consequently, under many probability measures over the space of preference profiles, including measures that allow for "local" correlation among agents' preferences, the probability that small coalitions of agents can successfully manipulate any scoring rule decreases to zero as the number of agents increases, establishing the "asymptotic strategyproofness" of scoring rules. Our results thus generalize the earlier result of Peleg (1979) that considered the same problem but restricted his attention to (generalized) rank order methods and independently and identically distributed agents' preferences.[1] We also show that for

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[1] Pazner and Wesley (1978) obtained a similar result for plurality rule.

Condorcet consistent rules, the proportion of manipulable preference profiles out of the set of preference profiles where a Condorcet winner exists decreases to zero at a rate proportional to $\frac{1}{\sqrt{n}}$ under the same distributional assumptions on agents' preferences. However, since the proportion of preference profiles where a Condorcet winner does not exist is bounded away from zero,[2] then depending on the way in which the chosen alternative is determined when a Condorcet winner does not exist, a Condorcet consistent rule may or may not be asymptotically strategyproof. We provide examples of asymptotically strategyproof and non strategyproof Condorcet consistent rules below.[3]

As Riker (1982) writes, the main problem with strategic voting is that it obscures the process of social choice which "may consist simply of the tastes of some people (whether a majority or not) who are skillful or lucky manipulators ... [outcomes] may consist of what the manipulators truly want, or they may be an accidental amalgamation of what the manipulators (perhaps unintentionally) happened to produce" (Riker, 1982, p. 167). Black (1958, p. 182) writes that when Borda was told that strategic voting could distort outcomes, he replied "My scheme is intended only for honest men." If agents have to incur a small cost in order to decide how to manipulate the voting rule, our results imply that all scoring rules and some Condorcet consistent rules are robust to strategic manipulation in large groups of agents. These voting rules can be expected to work well, regardless whether people are honest or not.[4]

A number of recent papers have compared the relative susceptibility to manipulation of different social choice functions (Chamberlin 1985; Kelly 1993; Lepelley and Mbih 1994; Nitzan 1985; Saari 1990; and Smith 1999). The results obtained here differ from those of the previous literature in three important respects: (1) As we explain in more detail below, we consider a more general model of correlation among agents' preferences, (2) whereas many of the previous results where based on numerical simulations or exhaustive counting, our results are analytic, and finally (3) we determine an upper bound on the rate of convergence to strategyproofness. This last result suggests that concern about the strategyproofness of scoring rules and some Condorcet consistent rules is of second-order importance even in environments with relatively small numbers of agents.

Our result also relate to a number of recent results in mechanism design theory that established an upper bound on the extent to which agents can be expected to

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[2] The literature on this is quite extensive, see, among others, Balasko and Crès (1997), DeMeyer and Plott (1970), Kelly (1974, 1986), Fishburn (1973), Gehrlein and Fishburn (1976), Moulin (1988, p. 230), and Tovey (1997).

[3] In another paper (Baharad and Neeman, 2000) we show that for *increasing* scoring rules, even for those profiles where a rule is manipulable, then as long as agents satisfy an appropriate "rationality" assumption, manipulation does not result in Pareto dominated alternatives being chosen. Inefficient manipulation may occur under non increasing scoring rules such as plurality rule, and under some Condorcet consistent rules.

[4] Moreover, the discussion below implicitly assumes that agents know other agents' preferences and how they behave under the voting rule. Such perfect knowledge is very rare in practice. Agents have to form beliefs over other agents' expected behavior, and decide, given their beliefs, what is the likely outcome and how best to manipulate it. Because of this uncertainty, the likelihood that manipulation will be successful is even lower than the bound we compute.

contribute to public good provision (Al-Najjar and Smorodinsky 2000; Lehrer and Neeman 2000; and Mailath and Postlewaite 1990) These results show that when the number of agents is large, the effect of any single agent on the probability of provision of a public good is negligibly small, and so the agents cannot be expected to contribute much to its production. These results may well be interpreted as establishing the asymptotic impossibility of manipulation.

The rest of the paper proceeds as follows. In the next section we describe the main assumptions about the joint distribution of agents' preferences. Section 3 is devoted to scoring rules, and Sect. 4 to Condorcet consistent rules.

2 The agents' preferences

We employ the following notation. Let A denote a finite set containing K different alternatives. Let $L(A)$ denote the set of linear orderings over A.[5] Let $N = \{1, ..., n\}$ denote a finite set of agents. For every $i \in N$, we denote i's preferences by $u^i \in L(A)$.

We focus our attention on environments where agents' preferences may be locally correlated.[6] That is, learning one agent's preference relation conveys some information about the preference relations of his "neighbors." Specifically, thinking of agents' preferences as random variables, we assume that the agents' preferences are ergodic random variables. For our purposes, it is enough to think of "ergodic" random variables, or preferences, as a general model of correlation among "close" members of a stationary sequence of random variables.[7] However, for the sake of completeness, we present a formal definition of ergodicity and provide a few examples below.

The definitions below are adapted from Durrett (1991). Call an event $E \subseteq \Omega$ invariant under the transformation $\varphi : \Omega \to \Omega$ if the set $\varphi^{-1}E = \{\omega : \varphi\omega \in E\}$ coincides with E up to a set of measure zero.

Definition. *A measure preserving transformation φ on the probability space (Ω, F, P) is ergodic if for every event $E \in F$ that is invariant under φ, $P(E) = 0$ or 1.*

Definition. *A sequence of random variables $V_1, V_2, ...$ that is generated by an ergodic measure preserving transformation φ such that for every $n \in \mathbb{N}$,*

$$V_n(\omega) = V_1\left(\varphi^{n-1}\omega\right)$$

for every $\omega \in \Omega$, is ergodic.[8]

[5] $L(A)$ contains all complete, transitive, and asymmetric preference profiles over A. Asymmetry implies that indifference is not allowed.

[6] The model of local correlation employed here is adapted from Lehrer and Neeman (2000).

[7] A sequence of random variables $X_1, X_2, ...$ is stationary if the joint distribution of $X_n, X_{n+1}, ..., X_{n+k}$ is identical to that of $X_1, ..., X_{k+1}$ for every two integers $n \geq 1$, and $k \geq 0$.

[8] In particular, ergodic random variables are also stationary.

Example 1. Every independent and identically distributed sequence of random variables is ergodic.

Example 2. Let $X_1, X_2, ...$ be a sequence of independent and identically distributed random variables with zero mean and unit variance. For every $n \in \mathbb{N}$, let $Y_n = \sum_{k=0}^{\infty} 2^{-k} X_{n+k}$. Note that $Y_1, Y_2, ...$ is a sequence of random variables with zero mean, a variance of $\frac{4}{3}$, and $\mathrm{cov}(Y_n, Y_{n+k}) = \frac{4}{3} \cdot 2^{-k}, k = 0, 1, 2, ...$ The sequence $Y_1, Y_2, ...$ is ergodic.[9]

Ergodicity implies that while agents' beliefs about their "neighbors' " preferences may be affected by their own preferences, in a large group, their "average" belief about the preferences of other agents in the group are almost independent by their own preferences. In particular, if the sequence of agents' preferences is such that they become more independent the farther they are apart as in Example 2 above, then the sequence is ergodic.[10]

We now define environments that exhibit local correlation. For every individual $i \in N$, and two different alternatives $a, b \in A$, define the random variable $p_{a,b}^i : L(A) \to \{-(K-1), -(K-2), ..., -1, 1, ..., K-1\}$ to be equal to $k - j$ if agent i with preferences u^i ranks alternative a above $k+1$ other alternatives and alternative b above $j+1$ other alternatives. We restrict our attention to the following type of environments.

Environments that exhibit local correlation. *Every agent $i \in N$ is equally likely to hold any preference $u^i \in L(A)$. In addition, for every pair of different alternatives $a, b \in A$,*

1. *the sequence $p_{a,b}^1, p_{a,b}^2, ...$ is ergodic, and*
2. $\sum_{m=2}^{\infty} \left(Var \left(E(p_{a,b}^m | p_{a,b}^1) \right) \right)^{\frac{1}{2}} < \infty.$

To understand the sense in which this definition implies that agents' preferences are locally correlated, note that condition 2 above implies that correlations between "remote" agents decrease to zero at a fast (enough) rate. For every two alternatives $a, b \in A$ and agent $m \in N$,

$$cov\left(p_{a,b}^1, p_{a,b}^m\right) = E\left[p_{a,b}^m p_{a,b}^1\right] - E\left[p_{a,b}^m\right] E\left[p_{a,b}^1\right].$$

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[9] See Durrett (1991, Thm. 1.3, p. 295.)

[10] The condition that agents' preferences become more independent the farther they are apart is known as mixing. It is a stronger condition than ergodicity that merely requires agents' preferences to become more independent of the average preference as the size of the group increases. Specifically, if a sequence of random variables is mixing, that is $\lim_{n\to\infty} P\left(A \cap \varphi^n B\right) = P(A)P(B)$ for all $A, B \in \mathcal{F}$ then it is ergodic. Conversely, if a sequence of random variables is ergodic then it is mixing "on average", that is $\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} P(A \cap \varphi^k B) = P(A)P(B)$ (see Durrett (1991) pp. 308-309).

Since, by assumption, every preference relation is equally likely, $E\left[p^m_{a,b}\right] = E\left[p^1_{a,b}\right] = 0$, therefore,

$$\begin{aligned} cov\left(p^1_{a,b}, p^m_{a,b}\right) &= E\left[p^m_{a,b}p^1_{a,b}\right] - E\left[p^m_{a,b}\right]E\left[p^1_{a,b}\right] \\ &= E\left[E\left[p^m_{a,b}p^1_{a,b}|p^1_{a,b}\right]\right] \\ &= E\left[p^1_{a,b}E\left[p^m_{a,b}|p^1_{a,b}\right]\right] \\ &\leq \left(E\left[\left(p^1_{a,b}\right)^2\right]\right)^{\frac{1}{2}}\left(E\left[\left(E\left[p^m_{a,b}|p^1_{a,b}\right]\right)^2\right]\right)^{\frac{1}{2}} \\ &= Var(p^1_{a,b})^{\frac{1}{2}}Var(E[p^m_{a,b}|p^1_{a,b}])^{\frac{1}{2}}. \end{aligned}$$

where the inequality follows from the Cauchy-Schwartz inequality. It therefore follows that $\sum_{m=2}^{\infty}\left(Var\left(E(p^m_{a,b}|p^1_{a,b})\right)\right)^{\frac{1}{2}} < \infty$ implies

$$\sum_{m=2}^{\infty} cov\left(p^1_{a,b}, p^m_{a,b}\right) < \infty.$$

This model of local correlation among agents' preferences is new to social choice literature which, for the most part, restricted its attention to two types of assumptions about the joint distribution of agents' preferences, the so called impartial culture and anonymous impartial culture assumptions.[11] Impartial culture implies that each agent is equally likely to hold every possible preference relation, and that different agents' preferences are mutually independent. Impartial anonymous culture implies that ordering all possible preference relations from 1 to $K!$ and letting n_1 denote the number of agents holding the first preference relation, n_2 denote the number of agents holding the second preference relation, and so on up to $K!$, then each possible vector $(n_1, n_2, ..., n_{K!})$ is equally likely. The assumption of anonymous impartial culture implies some (local) positive correlation among agents' preferences. This positive correlation is manifested through Gehrlein and Berg's (1992) characterization of different impartial culture assumptions in terms of symmetric Pólya-Eggenberger urn models. Consider an urn with $K!$ balls of different colors, each color corresponding to a different preference relation. The balls are drawn one at a time, and the color of the drawn ball determines the preferences of one agent. After each draw, the ball plus α additional balls of the same color are placed back into the urn. Obviously, the larger α, the higher the (local) positive correlation among agents' preferences. It is straightforward to verify that the assumption of impartial culture is equivalent to the special case where $\alpha = 0$; as Gehrlein and Berg (1992) show, impartial anonymous culture is equivalent to the special case where $\alpha = 1$.[12]

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[11] These assumptions are also standard in the more extensive literature that attempts to determine the proportion of profiles where a Condorcet winner exists that was mentioned in the introduction.

[12] We suspect that for every $\alpha \geq 0$, the generated sequence of preference relations is in fact ergodic, but are unable to verify this conjecture.

3 Scoring rules

Scoring rules are defined as follows (see Moulin, 1988, p. 231):

Definition: Scoring rules. *Fix a non decreasing sequence of real numbers* $s_0 \leq s_1 \leq \cdots \leq s_{K-1}$ *such that* $s_0 < s_{K-1}$. *The agents rank the alternatives, giving a score of* s_0 *to the alternative that is ranked last, a score of* s_1 *to the alternative that is ranked next to last, and so on until finally, the alternative that is ranked at the top is given the score* s_{K-1}. *The alternative that received the highest total score is selected. In case several alternatives all received the highest total score, an arbitrarily chosen tie-breaking rule may be used to determine the selected alternative from among those that received the maximum total score.*

As explained in the introduction, we seek to determine the robustness of scoring rules against strategic manipulation. Here and in the next section we define strategic manipulation as the act of misrepresenting one's true preferences in order to achieve a more favorable final outcome.

We have the following main result:

Proposition. *Suppose that agents' preferences exhibit local correlation as described above. Then, the probability that a finite coalition of fixed size can successfully manipulate any scoring rule decreases to zero at a rate proportional to* $\frac{1}{\sqrt{n}}$.

Proof. For every agent $i \in N$ and two alternatives $a, b \in A$, let $b^i_{a,b} : L(A) \to \{s_k - s_j\}_{\substack{0 \leq k,j \leq K-1 \\ k \neq j}}$ denote the difference between the score i assigns to a and b, respectively. For any $n \in \mathbb{N}$, and two different alternatives $a, b \in A$, let

$$B^n_{a,b}\left(u^1, ..., u^n\right) = \sum_{i=1}^{n} b^i_{a,b}(u^i)$$

denote the difference between the total scores of alternatives a and b.

For every $i \in \mathbb{N}$, and $a, b \in A$, the fact that every preference relation is equally likely implies that $E[b^i_{a,b}] = 0$. By assumption, $b^1_{a,b}, b^2_{a,b}, ...$ are ergodic random variables. Letting $\sigma^2 = Var(b^1_{a,b}) + 2\sum_{m=2}^{\infty} cov\left(b^1_{a,b}, b^m_{a,b}\right)$, a central limit theorem for dependent variables (see Durrett (1991, pp. 375–376)) implies,

$$\frac{B^n_{a,b}}{\sqrt{n}} \xrightarrow{d} N(0, \sigma^2).$$

Thus, for all large n, $\frac{1}{n}B^n_{a,b}$ is approximately normally distributed with mean zero and variance $\frac{\sigma^2}{n}$.

An agent can successfully manipulate a scoring rule and cause it to rank some alternative b above some other alternative a only if $0 \leq B^n_{a,b} \leq s_{K-1} - s_0$. Similarly, a coalition consisting of m agents can successfully manipulate a scoring rule only if $0 \leq B^n_{a,b} \leq m\left(s_{K-1} - s_0\right)$. Since $\frac{1}{n}B^n_{a,b}$ is approximately normally distributed, when there are $n+m$ agents and n is large, the probability of the latter event is given by

$$\begin{aligned}
&\Pr\left(0 \leq \frac{B_{a,b}^{n+m}}{n+m} \leq \frac{m}{n+m}\left(s_{K-1}-s_0\right)\right) \\
&\approx \frac{1}{\sigma\sqrt{\frac{2\pi}{n+m}}}\int_0^{\frac{m\left(s_{K-1}-s_0\right)}{n+m}} e^{-\frac{x^2}{\frac{2\sigma^2}{n+m}}}dx \\
&\leq \frac{\frac{m(s_{K-1}-s_0)}{n+m}}{\sigma\sqrt{\frac{2\pi}{n+m}}} \\
&= \frac{m\left(s_{K-1}-s_0\right)}{\sigma\sqrt{2\pi}}\frac{1}{\sqrt{n+m}} \underset{n\to\infty}{\searrow} 0
\end{aligned}$$

which, for a large n, decreases to zero at a rate proportional to $\frac{1}{\sqrt{n}}$. □

Numerical illustration. Consider Borda rule. That is, $s_k = k-1$ for $k \in \{1,...,K\}$. Suppose that there are $n+1$ agents whose preferences are independently and identically distributed. It is straightforward to verify that for every agent $i \in N$, two alternatives $a, b \in A$, and some $k \in \{1,...,K-1\}$, the number of profiles $u^i \in L(A)$ where $b_{a,b}^i(u^i) = k$ is $(K-2)!(K-|k|)$. Suppose first that $n=2$ and $K=3$. Notice that for every $k, j \in \{1,2,3\}$, the number of preference relations u^1, u^2 where agent 1's ranking is such that $b_{a,b}^1(u^1) = k$ and agent 2's ranking is such that $b_{a,b}^2(u^2) = j$ is equal to $(K-2)!\,(K-|k|)\,(K-2)!\,(K-|j|)$ which is equal to the coefficient of x^{k+j} in the expression

$$\left(x^{-2}+2x^{-1}+2x+x^2\right)^2.$$

More generally, because agent i may successfully manipulate in favor of alternative a over alternative b only if $-(K-1) \leq B_{a,b}^n \leq 0$.[13] When the number of agents is $n+1$ and the number of alternatives is $K \geq 2$, the proportion of profiles where an agent can successfully manipulate the outcome under Borda rule is given by the sum of the coefficients of $x^{-(K-1)}, x^{-(K-2)}, ..., x^0$ in the expansion of the expression

$$\Big((K-2)!x^{-(K-1)} + 2(K-2)!x^{-(K-2)} + \cdots + (K-1)!x^{-1} \\
+(K-1)!x^1 + \cdots + (K-2)!x^{K-1}\Big)^n$$

over the total sum of coefficients. Calculations using MAPLE show that the proportion of profiles where agents can successfully manipulate as a function of n and K, are:

[13] It is enough to consider this case since when $B_{a,b}^n > 0$, alternative a is preferred over b anyway.

n	$K=3$	$K=4$	$K=5$
6	22.5%	25.1%	27.0%
10	16.5%	19.9%	21.5%
30	10.1%	11.8%	⋮
50	7.9%	⋮	
⋮	⋮	⋮	⋮
∞	0	0	0

4 Condorcet consistent rules

A rule is Condorcet consistent (see, e.g., Moulin, 1988, p. 229) if it selects the Condorcet winner (an alternative that is preferred to every other alternative by a majority of the agents) whenever it exists. As indicated in the introduction, among all the preference profiles where a Condorcet winner exists, the fraction of profiles where a coalition of size m can successfully manipulate a Condorcet consistent rule decreases to zero at a rate proportional to $\frac{1}{\sqrt{n}}$. The proof is similar to the proof of the Proposition above.

For every individual $i \in N$, and two different alternatives $a, b \in A$, define the random variable $\widehat{p}^i_{a,b} : L(A) \to \{-1, 1\}$ to be equal to 1 if i prefers a to b under u^i, and to -1 otherwise (recall that indifference is not allowed). Note that

$$M^n_{a,b}\left(u^1, ..., u^n\right) = \sum_{i=1}^{n} \widehat{p}^i_{a,b}\left(u^i\right)$$

determines the relative social ranking of alternatives a and b according to majority rule. As before, we assume that the sequence $\widehat{p}^1_{a,b}, \widehat{p}^2_{a,b}, ...$ is ergodic and that the correlation among agents' preferences decreases to zero the farther they are apart, or $\sum_{m=2}^{\infty}\left(Var\left(E(\widehat{p}^m_{a,b}|\widehat{p}^1_{a,b})\right)\right)^{\frac{1}{2}}$ is finite. Our assumptions imply that for large n, $\frac{1}{n}M^n_{a,b}$ is approximately normally distributed with zero mean and a finite variance. The rest of the argument follows from the fact that a coalition of m agents can successfully manipulate majority rule and cause it to rank alternative b above alternative a only if $0 \leq M^n_{a,b} \leq m$. An argument similar to the one presented in the proof of the proposition shows that the probability of this event decreases to zero at a rate proportional to $\frac{1}{\sqrt{n}}$ when n is large.

Thus, since as mentioned in the introduction, the proportion of preference profiles where a Condorcet winner fails to exist is bounded above zero, whether a Condorcet consistent rule is asymptotically strategyproof depends on the way in which the winning alternative is determined when a Condorcet winner does not exist. Below, we present three examples: two of a Condorcet consistent rule that is asymptotically strategyproof, and one that is not.

Asymptotically strategyproof Condorcet consistent rules – the top cycle and Copeland rules. The top cycle rule selects the alternative that is preferred by a majority of the agents to all other alternatives if such an alternative exists. When such a Condorcet winner does not exist, the top cycle rule selects randomly from among the top-cycle (the transitive closure of majority rule). It is formally defined as follows (see Moulin, 1988, p. 253).

Top cycle rule. *For any two different alternatives $a, b \in A$, let $a \succeq_T b$ if and only if there is an integer q and a sequence $a = a_0, a_1, ..., a_q = b$, such that a_j is preferred to a_{j+1} by at least half of the agents for every $j \in \{0, 1, ..., q-1\}$. The top cycle is defined as the non empty set of maximal elements of T. Namely, an alternative a belongs to the top cycle if and only if $a \succeq_T b$ for every alternative $b \neq a$. The top cycle rule selects each alternative in the top cycle with equal probability.*

The Copeland rule selects the alternative that defeats the largest number of other alternatives. It is formally defined as follows (see Moulin, 1988, p. 233).

Copeland rule. *For any alternative $a \in A$, compare alternative a with any other alternative $b \in A$. Score $+1$ if a majority prefers a to b, -1 if a majority prefers b to a, and 0 if the two alternatives tie. Summing up the score of a over all $b \in A$ yields the Copeland score of a. The alternative with the highest Copeland score is selected. In case several alternatives all received the highest Copeland score, every such alternative is selected with equal probability.*

The reason that both rules are asymptotically strategyproof is that under both rules, small coalitions of agents cannot manipulate the decision of the majority of the agents, from which it follows that they cannot manipulate when a Condorcet winner does not exist either.

A Condorcet consistent rule that is not asymptotically strategyproof. Suppose there are only three alternatives, a, b, and c, and that the number of agents is odd. Consider the following rule: select the Condorcet winner whenever it exists. When it does not exist, it must be that the top cycle includes alternatives a, b, and c. Then, select alternative a if the number of agents who ranked a as their most favored alternative is even, and select alternative b otherwise. It is straightforward to verify that on all those profiles where a Condorcet winner does not exist (which according to Gehrlein and Fishburn, 1976 tends to a limit of 8.8% of all possible profiles), even a single agent can manipulate in favor of alternative a or b (or both).

References

Al-Najjar, N. I., Smorodinsky, R. (2000) Pivotal players and the characterization of influence. *Journal of Economic Theory* 92: 318–342

Baharad, E., Neeman, Z. (2000) The Robustness of Scoring Rules Against Inefficient Manipulation. Mimeo, Boston University (http://people.bu.edu/zvika/)

Balasko, Y., Crès, H. (1997) The probability of Condorcet cycles and super majority rules. *Journal of Economic Theory* 75: 237–270

Black, D. (1958) *The theory of committees and elections.* Cambridge University Press, Cambridge

Chamberlin, J.R. (1985) An investigation into the relative manipulability of four voting systems. *Behavioral Science* 30: 195–203

DeMeyer, F., Plott, C.R. (1970) The probability of a cyclical majority. *Econometrica* 38: 345–354

Durrett, R. (1991) *Probability: Theory and examples*. Wadsworth and Brooks/Cole, Pacific Grove, Ca

Fishburn, P.C., Gehrlein, W.V. (1980) Social homogeneity and Condorcet's paradox. *Public Choice* 35: 403–419

Gehrlein, W. V., Fishburn, P.C. (1976) The probability of the paradox of voting: a computable solution. *Journal of Economic Theory* 13: 14–25

Gehrlein, W.V., Berg, S. (1992) The effect of social homogeneity on coincidence probabilities for pairwise proportional lottery and simple majority rules. *Social Choice and Welfare* 9: 361–372

Kelly, J.S. (1974) Voting anomalies, the number of voters, and the number of alternatives. *Econometrica* 42: 239–251

Kelly, J.S. (1986) Condorcet winner proportions. *Social Choice and Welfare* 3: 311–314

Kelly, J.S. (1993) Almost all social choice rules are highly manipulable, but a few aren't. *Social Choice and Welfare* 10: 161–175

Lepelley, D., Mbih, B. (1994) The vulnerability of four social choice functions to coalitional manipulation of preferences. *Social Choice and Welfare* 11: 253–265

Lehrer, E., Neeman, Z. (2000) The scope of anonymous voluntary bargaining under asymmetric information. *The Review of Economic Studies* 67: 309–326

Mailath, G.J., Postlewaite, A. (1990) Asymmetric information bargaining problems with many individuals. *Review of Economic Studies* 57: 351–367

Moulin, H. (1988) *Axioms of cooperative decision making*. Econometric Society Monograph, Cambridge University Press

Nitzan, S. (1985) The vulnerability of point-scoring schemes to preference variation and strategic manipulation. *Public Choice* 47: 349–370

Pazner, E., Wesley, E. (1978) Cheatproofness properties of the plurality rule in large societies. *Review of Economic Studies* 45: 85–91

Peleg, B. (1979) A note on manipulability of large voting schemes. *Theory and Decision* 11: 401–412

Riker, W. (1982) *Liberalism against populism*. Freeman, San Francisco

Saari, D.G. (1990) Susceptibility to manipulation. *Public Choice* 64: 21–41

Smith, D.A. (1999) Manipulability measures of common social choice functions. *Social Choice and Welfare* 16: 639–661

Tovey, C.A. (1997) Probabilities of preferences and cycles with super majority rules. *Journal of Economic Theory* 75: 271–279