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# The Scope of Anonymous Voluntary Bargaining Under Asymmetric Information

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We present a model of anonymous collective bargaining where individuals' preferences and information may be significantly interdependent. We show that the bargaining outcome becomes independent of individuals' preferences and information as the bargaining group increases in size. As a corollary, we show that anonymous voluntary bargaining completely fails in large groups. Either the difference between the bargaining outcome and the status quo vanishes as the size of the group becomes larger, or, the bargaining becomes coercive and results in a violation of at least some individuals' rights. The result provides a rationale for the inherent difficulty of reform in the presence of asymmetric information.

"There is nothing more difficult to carry out, nor more doubtful of success, nor more dangerous to handle, than to initiate a new order of things."

Niccolò Machiavelli, *The Prince* (1532)

## 1. INTRODUCTION

A group of individuals is contemplating a change that will affect their utilities. In order to facilitate the proposed change, the group may administer monetary transfers among its members. Many collective choice problems conform to this structure: the provision of public goods, the enactment of property rights' reforms, the construction of a production facility that generates pollution and more.[1] If individuals' utilities before and after the change are commonly known, it is not difficult to devise a system of monetary transfers that secures an increase in each individual's utility when such a system exists. The reasoning underlying the Coase Theorem (Coase (1960)) implies that if there are no transaction costs, the group will be able to successfully solve its bargaining problem, implement change whenever it is welfare enhancing, and improve the situation of all its members. The bargaining problem becomes considerably more difficult when the individuals are asymmetrically informed about the difference in their utilities as a result of the proposed change. In order to arrange a system of transfers that secures increases in utilities, the group must induce individuals to truthfully reveal their private information. We show that this becomes increasingly more difficult, and asymptotically impossible to achieve, as

1. See Mailath and Postlewaite (1990) for additional examples.

the group increases in size. As a consequence, the decisions about whether to introduce the proposed change, and about the sizes of the monetary transfers if change is implemented are completely arbitrary; they are necessarily made independently of the change in individuals' utilities. A corollary of the above is that if, in addition, bargaining is constrained to be voluntary in the sense that individuals may veto any decision that does not increase their expected true utility, and the proposed change is such that some individuals might suffer as a result, then change is impossible in large groups, even if it significantly enhances total welfare.

We derive this result in a model where individuals are endowed with general utility functions and where information, beliefs, and preferences are interdependent. We focus on the case where individuals' information may be highly correlated with that of their "neighbours" but almost independent of the information of more remote individuals. We thus have a model with "local" correlation but "global" independence. We consider a sequence of bargaining problems as described above where the groups are increasing in size. We impose two conditions on this sequence of bargaining problems. First, bargaining must be "anonymous," and second, the sequence of bargaining outcomes must be "consistent." Anonymity requires invariance with respect to individuals' identities. It implies that the bargaining outcome can only depend on the distribution of (reported) preferences in the group, not on individuals' identities. Consistency requires decisions based on large enough samples of the population to be similar to the decision in the limit group that consists of a countably infinite number of individuals. We show that for any sequence of groups increasing in size, the outcome obtained by any sequence of incentive compatible, anonymous, and consistent bargaining procedures, converges to a constant that is independent of individuals' information. This implies that *voluntary* bargaining completely fails in large groups. Either the difference between the outcome of the bargaining process and the status quo vanishes as the size of the group becomes larger, or the process of bargaining becomes coercive and violates at least some individuals' rights.

We thus propose an explanation for the difficulty that is intrinsic to accomplishing change. Any (non-trivial) proposed change produces individuals that gain and individuals that lose with respect to the status quo. A reasonable decision rule is to implement change if and only if it increases total welfare, and then to tax those who gain and compensate the losers. This gives "gainers" an incentive to free-ride by understating the extent of their gains, while losers are encouraged to free-ride by exaggerating the extent of their losses. The only countervailing incentive to report truthfully is that lying increases the chances that change will not take place and the positive utility individuals could gain if change were implemented, will be lost. When the group becomes larger, this countervailing incentive becomes correspondingly weaker because the effect of any single individual on the group's decision becomes negligible, so the tendency to misrepresent one's true preferences prevails. Similar reasoning shows that other decision rules will also fail to aggregate individuals' information.

Our explanation suggests that change is more difficult to achieve when the number of individuals involved in the bargaining process is large, and when the uncertainty regarding their costs and benefits is considerable. Libecap (1989) presents several case studies of property rights reforms in the U.S. and shows that both factors indeed play a significant role in determining the speed and efficacy of these reforms.[2]

2. The most striking example is the case of the reform in the property rights for oil. Petroleum was first discovered in the United States in 1859. Libecap and Wiggins (1985) report that as late as 1975, more than a century later, as many as 60% of the oil fields in Oklahoma and 80% in Texas were still not completely unitized in spite of the general agreement about the efficiency of unitization. (See also Wiggins and Libecap (1985).)

The negative result of this paper stands in sharp contrast to the positive results of the mechanism design under correlated information literature, (Crémer and McLean (1985, 1988), Johnson, Pratt and Zeckhauser (1990) and McAfee and Reny (1992)), that show that even the slightest correlation among individuals' information allows for the implementation of virtually any decision rule. These results depend on the assumption that there exists a *publicly known* one-to-one correspondence between individuals' preferences and individuals' beliefs. This enables the construction of a menu of lotteries that induces individuals to report their information truthfully and allows for the extraction of their entire "informational rents." We believe that while information about the distribution of individuals' information and preferences may be publicly available through statistical surveys for example, more precise information that includes individuals' identities is increasingly more difficult to obtain. Specifically, while information regarding individual $i$'s beliefs about the distribution of preferences in the economy may be revealed by individual $i$'s report of specific preferences and beliefs, it is unlikely that the mechanism designer will be able to form a consistent prior over individual $i$'s beliefs regarding a particular individual $j$.

More generally, the point is that mechanism design literature does not distinguish between the different components of individuals' types. By reporting their *preferences*, individuals also reveal their *beliefs* and vice versa. We contend that there ought to be some degree of independence between individuals' beliefs and individuals' preferences.[3] The assumption of anonymity is respectful of this distinction. In determining individual $i$'s payment, it forces the mechanism designer to rely only on the correlation between $i$ and the entire group. The assumption that the correlations between individuals' types and global indices are decreasing then implies that the type of lotteries used by Crémer and McLean (1985, 1988) and McAfee and Reny (1992) become more difficult to construct in large groups. As a consequence, not only is voluntary bargaining becoming more difficult in the sense that it cannot implement the efficient outcome, but it cannot implement any deviation from the status quo whatsoever.

In addition, since non-anonymous mechanisms require keeping track of the position of every individual with respect to every other individual, complexity considerations are likely to render non-anonymous mechanisms in large groups too costly to be of practical interest. Thus, we believe that the generality of our conclusions is not seriously compromised by restricting our attention to anonymous bargaining. However, we recognize the fact that this general interpretation may appear as unjustified to some readers. Therefore, in light of the wide use and effectiveness of anonymous mechanisms in symmetric environments,[4] we emphasize that our results may also lend themselves to a more modest interpretation; namely, as highlighting the limitations of relying only on anonymous mechanisms.

The rest of the paper proceeds as follows. In the next section, we present the model and state the results. We provide the proofs in Section 3. Section 4 offers discussion of the result and the related literature. Section 5 concludes with a discussion of the relevance of our results to the political economy literature. An Appendix contains an illustration of our main result in the context of a specific example.

3. For further development of this idea, see Neeman (1997).

4. For example, in a recent paper, Al-Najjar and Smorodinsky (1997) have shown that anonymous mechanisms in symmetric environments maximize the players' average "influence" with respect to all other mechanisms and environments.

## 2. THE MODEL AND STATEMENT OF THE RESULTS

A group of $n$ individuals has to decide whether to undertake an action that will create a change in their utilities. The "level" or "degree" of change is denoted by $q \in [0, 1]$. A choice of $q=0$ indicates no change and continuation of the status quo situation. The utility of each individual $i$ in the group depends on the degree of change $q$, the net monetary transfer she receives $t_i$, and a parameter $v_i \in \mathscr{V}$, and is given by the function $u(q, t_i, v_i)$. Individuals' utility functions are assumed to be continuous, strictly increasing in the size of the monetary transfer and strictly monotonic in the degree of change so that $u(\cdot, t_i, v_i)$ is either increasing or decreasing in the degree of change $q$. Different individuals may have different preferences depending on their type $v_i \in \mathscr{V}$, where $\mathscr{V}$ is a countable set of real numbers. We refer to $v_i$ as individual $i$'s type, information, valuation, or preferences, interchangeably. The monotonicity of $u$ with respect to $q$ implies that individuals, depending on their type, may either benefit or suffer from the proposed change. We normalize individuals' utilities such that their status quo utility, absent any monetary transfers, is zero regardless of their type. That is, $u(0, 0, v_i) = 0$ for all $v_i \in \mathscr{V}$. Finally, we assume that there is at least one type of individual who will benefit from the proposed change if it is implemented, and one type of individual who will suffer from the proposed change if it is implemented. Otherwise, it is common knowledge that either the status quo ($q=0$) or the most radical change ($q=1$) wins the unanimous support of the group.

We study the performance of rules or institutions that determine the degree of change. These institutions may mandate compensatory monetary transfers in addition to determining the degree of change. By the revelation principle (see, *e.g.* Myerson (1985)) the decision obtained under any such rule can be represented by the truth-telling equilibrium of an incentive compatible direct revelation mechanism. A direct revelation mechanism consists of a decision function $q_n : \mathscr{V}^n \to [0, 1]$ that maps individuals' reports of their types into a decision about the degree of change and $n$ functions $t_n^i : \mathscr{V}^n \to \mathbb{R}^n$ that map individuals' reports into their monetary transfers.

We model the bargaining process through which the degree of change and the monetary transfers are determined as a Bayesian game. Individuals' types are given by a sequence of random variables $V_1, V_2, \ldots$ that are defined on a probability space $(\Omega, \mathscr{F}, P)$. At each state of the world $\omega$, each individual $i$ knows her type $V_i(\omega) = v_i$. Her belief about other individuals' types is given by her conditional expectation, $P(\cdot | V_i(\omega) = v_i)$. We ensure that individuals' beliefs are well defined by assuming that $P(V_i(\omega) = v_i)$ is positive for all $i$ and $v_i \in \mathscr{V}$.

We are interested in modelling a situation where individuals' types are "locally" correlated but "globally" independent. Individuals may be quite knowledgeable about their friends and neighbours (in the sense that their posterior beliefs about their friends' types are very different from the prior belief), but since a lot of credible public information is likely to be available about global indices such as the average type in the group, it is unlikely that individuals would hold drastically different opinions on these matters. We capture this correlation structure in our model by assuming that individuals' types, that is, the random variables $V_1, V_2, \ldots$ are generated by a *covering transformation* $\varphi : \Omega \to \Omega$ in the following way: $V_1$ is given, and for every $n \in \mathbb{N}$,

$$V_n(\omega) = V_1(\varphi^{n-1}\omega) \quad \text{for every } \omega \in \Omega. \tag{*}$$

*Definition.* A transformation $\varphi$ on $(\Omega, F, P)$ is covering if for every event $E \in F$, $P(E) > 0$ implies that:

(a) there is an $n \geq 1$ such that $P(\varphi^{-n}E) > 0$;
and
(b) $P(\bigcup_{n \geq 1} \varphi^n E) = 1$.

The assumption that individuals' types are generated by a covering transformation guarantees that individuals' beliefs about the distribution of other individuals' types are likely to be heterogeneous both in the sense that individuals may have widely different beliefs about different individuals and in the sense that different individuals may have widely different beliefs about the same individual. The following series of examples illustrate the range of priors that can and cannot be generated by a covering transformation and explains the way in which covering is related to heterogeneity, local correlation, and global independence.

*Example* 1. The simplest non-trivial example is where $\Omega = \{\omega_1, \omega_2\}$, $\mathcal{F} = 2^{\Omega}$, and $0 < P(\omega_1) < 1$. There are four possible transformations: (i) the identity transformation, $\varphi(\omega_i) = \omega_i$, $i \in \{1, 2\}$; (ii) a transformation that switches between $\omega_1$ and $\omega_2$, $\varphi(\omega_i) = \omega_j$, $i \neq j \in \{1, 2\}$; (iii) a transformation into $\omega_1$, $\varphi(\omega_i) = \omega_1$, $i \in \{1, 2\}$; and (iv) a transformation into $\omega_2$, $\varphi(\omega_i) = \omega_2$, $i \in \{1, 2\}$; Of these four transformations, only the one that switches between $\omega_1$ and $\omega_2$ is covering. To see the relationship between covering and heterogeneity, note that if in addition $V_1(\omega_1) \neq V_1(\omega_2)$, then in case (ii) where $\varphi$ is covering, half the population has one type and the other half the other type. On the other hand, if $\varphi$ is not covering, then in case (i) all individuals have the same type and in cases (iii) and (iv) while there is some uncertainty about individual 1's type, individuals $2, 3, \ldots$ have the same commonly known type.

Perhaps the most heterogeneous example of a sequence of random variables is the following.

*Example* 2. Every independent and identically distributed sequence of random variables can be generated by a covering distribution.

Heterogeneity can also co-exist with correlation.

*Example* 3. Let $X_1, X_2, \ldots$ be any sequence of independent and identically distributed random variables with zero mean and unit variance. For every $n \in \mathbb{N}$, let $Y_n = \sum_{k=0}^{\infty} 2^{-k} X_{n+k}$. Note that $Y_1, Y_2, \ldots$ is a sequence of random variables with zero mean, a variance of $\frac{4}{3}$, and $\operatorname{cov}(Y_n, Y_{n+k}) = \frac{4}{3} \cdot 2^{-k}$, $k = 0, 1, 2, \ldots$ The sequence $Y_1, Y_2, \ldots$ can be generated by a covering transformation.

Examples 2 and 3 above are particular cases of a more general class of random variables that can be generated by a covering transformation, the class of ergodic random variables.

*Example* 4. Every ergodic sequence of random variables can be generated by a covering transformation. In fact, ergodic random variables are the general class of random variables that can be generated by measure preserving covering transformations.[5] Ergodicity implies that while individuals' beliefs about their "neighbours'" types may be affected

5. The standard definition of ergodic random variables is the following. Call an event $E \in \mathcal{F}$ invariant under $\varphi$ if the set $\varphi^{-1}E = \{\omega : \varphi\omega E\}$ coincides with $E$ up to a set of measure zero.

*Definition.* A measure preserving transformation $\varphi$ on $(\Omega, F, P)$ is ergodic if for every event $E \in F$ that is invariant under $\varphi$, $P(E) = 0$ or 1.

*Definition.* A sequence of random variables $V_1, V_2, \ldots$ that is generated by an ergodic measure preserving transformation $\varphi$ (as in (*)) is ergodic.

by their own types or preferences, their "average" belief about the preferences of other individuals in the group remains almost entirely unaffected. In particular, if the sequence of individuals' types is such that individuals' types become more independent the farther they are apart as in Example 3 above, then the sequence is ergodic.[6]

The difference between ergodic sequences of random variables and other sequences of random variables that can be generated by a covering transformation is that the latter need not be stationary, or identically distributed. (Note that by (*), ergodic sequences are stationary.) We illustrate this in the following example.

*Example* 5. Let $V_1, V_2, \ldots$ be a sequence of random variables that are generated by $\varphi$, a measure preserving ergodic transformation on $(\Omega, \mathscr{F}, P)$. Assume that $Q$ is a probability distribution such that $P$ and $Q$ are mutually absolutely continuous. That is, for every event $E \in \mathscr{F}$, $Q(E) > 0$ if and only if $P(E) > 0$. Since $\varphi$ is covering with respect to $P$, it is also covering with respect to $Q$, but the sequence $V_1, V_2, \ldots$ need not necessarily be stationary with respect to $Q$.

To better understand the range of types of sequences of random variables that can be generated by a covering transformation, it is useful to present a class of stationary sequences of random variables that cannot be generated by a covering transformation.

*Example* 6. A class of stationary sequences of random variables that cannot be generated by a covering transformation is the class of exchangeable random variables. A simple example of one such sequence was already described in Example 1, case (i), above. Namely, the case of a stationary sequence of random variables $V_1, V_2, \ldots$ that with probability $p$ are all equal to some constant $c_1$ and with probability $1-p$ are all equal to some other constant $c_2 \neq c_1$. More generally, any mixture of i.i.d. random variables is exchangeable and cannot be generated by a covering transformation. By de Finetti's Theorem (see, *e.g.* Durrett (1991, p. 232)) an exchangeable sequence of random variables is conditionally i.i.d. That is, there exists a random variable $X$ such that conditional on $X$, $V_1, V_2, \ldots$ are i.i.d. Thus, by focusing on ergodic sequences we focus our attention on situations in which if indeed such a random variable $X$ exists, then it is known to the individuals as well as to the mechanism designer.

Relying on the revelation principle implies that mechanisms are required to be incentive compatible. Individuals must be induced to honestly report their preferences when they believe that all other individuals do.

*Definition.* A mechanism $\langle q_n, t_n \rangle$ is incentive compatible if

$$E[u(q_n(V), t_n^i(V), V_i) \mid V_i(\omega) = v_i] \geq E[u(q_n(\hat{v}_i, V_{-i}), t_n^i(\hat{v}_i, V_{-i}), V_i) \mid V_i(\omega) = v_i]$$

for all $i \in \{1, \ldots, n\}$ and $v_i, \hat{v}_i \in \mathscr{V}$.

6. The fact that every sequence of ergodic random can be generated by a measure-preserving covering transformation follows from the fact that by (*) every ergodic sequence of random variables is stationary, and every stationary sequence of random variables can be generated by a measure-preserving transformation. The proof is simple and is by construction. See, *e.g.* Petersen (1983, pp. 6–7). Standard arguments imply that the sequences of random variables described in Examples 2 and 3 and example described in the Appendix are ergodic. (See, *e.g.* Durrett (1991, Thm. 1.3, p. 295).)

We employ standard notation, namely, $V=(V_1,\ldots,V_n)$, $V_{-i}=(V_1,\ldots,V_{i-1},V_{i+1},\ldots,V_n)$ and $(v_i,V_{-i})=(V_1,\ldots,V_{i-1},v_i,V_{i+1},\ldots,V_n)$. Note that because individuals' types may potentially be correlated, their types provide them with information about the distribution of other individuals' types from which they can infer information about the mechanism's decisions. Therefore, their expected utilities are conditional on their types.

We also require the mechanism to be feasible. The compensatory transfers must be financed from within the group. This implies that the sum of the monetary transfers cannot be positive. We impose a weaker condition that only requires the expected sum of the monetary transfers to be nonpositive.[7]

*Definition.* A mechanism $\langle q_n, t_n\rangle$ is feasible if

$$E[\textstyle\sum_{i=1}^n t_n^i(V)]\leq 0.$$

We consider an increasing sequence of groups, $\{G_n\}$, that are bargaining over a proposed change. Each group $G_n$ consists of the $n$ individuals $\{1,\ldots,n\}$. Thus, individual $i$'s beliefs about individual $j$'s type are independent of the particular group in which they interact. We require the bargaining process to be anonymous as follows.

*Definition.* A mechanism $\langle q_n, t_n\rangle$ is anonymous if $q_n(v_1,\ldots,v_n)=q_n(v_{\pi_n(1)},\ldots,v_{\pi_n(n)})$ for all vectors $v\in\mathscr{V}^n$, and permutations $\pi_n\colon\{1,\ldots,n\}\to\{1,\ldots,n\}$ and $t_n^i(v_1,\ldots,v_n)=t_n^i(v_{\pi_n(1)},\ldots,v_i,\ldots,v_{\pi_n(n)})$ for all $i\in\{1,\ldots,n\}$, vectors $v\in\mathscr{V}^n$, and permutations $\pi_n^i\colon\{1,\ldots,n\}\to\{1,\ldots,n\}$ that leave $i$ unchanged.

Anonymity requires the mechanisms' decisions to be invariant with respect to individuals' identities. Thus, it implies that the mechanisms' decisions can only depend on the distribution of individuals' types. Note, however, that individual $i$'s transfer may well depend on her report. It is symmetric only with respect to other individuals' reports. Anonymity entails no loss of generality if the central planner is uncertain about individuals' identities and considers each permutation of individuals' identities to be equally likely. To see this, consider a mechanism $\langle q_n, t_n\rangle$ that is not necessarily anonymous. Suppose that the permutations from $\{1,\ldots,n\}$ into itself are given by $\pi_1,\ldots,\pi_{n!}$. From the planner's perspective, if the realization of individuals' types is $(v_1,\ldots,v_n)$, the outcome is $q_n(v_{\pi_i(1)},\ldots,v_{\pi_i(n)})$ with probability $1/n!$ for every $i\in\{1,\ldots,n!\}$. The same distribution of outcomes can be obtained by the anonymous mechanism that picks a permutation $\pi_i$ at random and then applies $q_n$. A similar argument can be used to show that $t_n$ can be required to be anonymous without further loss of generality.[8,9]

Given a mechanism $\langle q_n, t_n\rangle$, define the random variables $Q_n(\omega)=q_n(V_1(\omega),\ldots,V_n(\omega))$ and $T_n^i(\omega)=t_n^i(V_1(\omega),\ldots,V_n(\omega))$. $Q_n$ is the random variable that describes the degree of

7. This weaker condition applies if the group has access to well-functioning credit markets where it can insure itself against bankruptcy in return for the surplus generated through the transfers.

8. This argument shows that *ex ante* anonymity is without loss of generality, whereas our definition of anonymity is an *ex post* concept. The apparent discrepancy disappears if we extend our definition of mechanisms to be from individuals' reports into the space of probability distributions over outcomes. Our results do not depend on which definition is used.

9. It should be noted that, as explained in the introduction, this argument fails to work if the planner can ask individuals to report their beliefs, or identities, as well as their types $v_i$. Of course, the planner will have to make sure that truthful reporting of beliefs is incentive compatible. However, a system that files, contrasts, and compares such reports is likely to be very expensive to operate.

change accomplished by the group $G_n$ under the mechanism $\langle q_n, t_n \rangle$ when everyone is reporting their true valuations, and $T_n^i$ is the random variable that describes individual $i$'s monetary transfer when she reports $V_i(\omega) = v_i$ under the mechanism $\langle q_n, t_n \rangle$ used by the group $G_n$ when everyone is truthful.

*Definition.* A sequence of mechanisms $\{\langle q_n, t_n \rangle\}$ is consistent if for every $\varepsilon > 0$ there exists an $N$ such that $P(|Q_n - Q_m| > \varepsilon) < \varepsilon$ for every $n, m \geqq N$; and, for every $i \in N$ and $\varepsilon > 0$ there exists an $N$ such that $P(|T_n^i - T_m^i| > \varepsilon) < \varepsilon$ for every $n, m \geqq N$.

The assumption of consistency is the analogue of the Cauchy condition for a sequence of real numbers to a sequence of real functions. It implies that small groups of individuals are effective in the following sense:[10] For any pre-specified level of precision, there exists a large enough group such that the decision made in this group is close to the decisions made in larger groups with a high probability. Because the groups are embedded within each other it implies that for any pre-specified level of precision, there exists a large enough $n$ such that the input of individuals $\{1, \ldots, n\}$ determines the outcome in the groups that consist of individuals $\{1, \ldots, n+k\}$ for any integer $k \geqq 1$, up to the pre-specified degree of precision, with a high probability. Alternatively, consistency implies that in the limit group (that consists of a countable number of individuals), if, say, in order to minimize costs, a decision is made to base the decision about the appropriate degree of reform on a finite sample of the population, then for any pre-specified level of precision, basing the decision on a large enough sample of individuals $\{1, \ldots, n\}$ approximates the correct decision with a high probability. Formally, consistency is required in order to guarantee convergence (in probability) of a subsequence of the sequences $\{Q_n\}$ and $\{T_n\}$. It reflects the fact that "similar" mechanisms are employed by all large enough groups.[11] We leave further discussion of the role of the consistency assumption to Section 4.2.

Finally, in order to guarantee the existence (finiteness) of individuals' expected utilities in the limit group, we require the mechanisms to be admissible in the following sense.

*Definition.* A mechanism $\langle q_n, t_n \rangle$ is admissible if $|T_n^i|$, $i \in \{1, \ldots, n\}$, are bounded by a random variable $\bar{T}^i(V_i)$ (that may possibly depend on $i$ and $v_i \in \mathscr{V}$), such that both $E[u(1, \bar{T}^i(v_i), v_i)]$ and $E[u(1, -\bar{T}^i(v_i), v_i)]$ are finite.

Note that although $|T_n^i|$ is uniformly bounded by another random variable, its realization may still be arbitrarily large.

We are now in a position to present our main result. Its proof is relegated to the next section.

**Theorem.** *Let $\{G_n\}$ be a sequence of groups of individuals whose types are given by a sequence of random variables that are generated by a covering transformation. Each group $G_n$ adopts a feasible, incentive compatible, and anonymous mechanism $\langle q_n, t_n \rangle$ to bargain over a proposed change, and the sequence of mechanisms $\{\langle q_n, t_n \rangle\}$ is consistent and admissible. It follows that the degree of change that is acccomplished by the group $G_n$ and individuals'*

10. Wooders (1994) employs a different "small groups are effective" condition in a different context.

11. Consistency is satisfied if, for example, the $\langle q_n, t_n \rangle$s are functions of the sum of individuals' reports. A stronger condition that implies consistency requires the sequence of mechanisms $\{\langle q_n, t_n \rangle\}$ to be equicontinuous and uniformly bounded. Consistency then follows from the Ascoli–Arzelá Theorem (see, *e.g.* Royden (1988, pp. 167–169)).

*monetary transfers converge to constants in probability. That is, $Q_n \to c$ in probability for some $c \in [0, 1]$ and $T^i_n \to t^i$ in probability for some $t^i \in \mathbb{R}$ for all $i \in \mathbb{N}$.*

The fact that the degree of change and individuals' monetary transfers converge to constants implies that, in the limit, they must become independent of individuals' information and preferences. By itself, the fact that the degree of change converges to a constant does not imply that it is not responsive to individuals' information and preferences, because the distribution of preferences in the groups converges as well. Thus, if the (*ex ante*) distribution of individuals' preferences is such that change is favourable on average, in large groups implementing change will almost surely increase total welfare. However, the fact that the monetary transfers converge to constants as well implies that "large" groups cannot administer a system of compensatory transfers that will enable all individuals to share the benefits that are generated by the implemented change.

Define bargaining to be voluntary if individuals are not coerced into agreeing with its outcomes. In particular, every individual has the right to veto any decision that provides her with a utility that is lower than her utility under the status quo.[12] A voluntary bargaining mechanism must therefore be individually rational.

*Definition.* A mechanism $\langle q_n, t_n \rangle$ is individually rational if

$$E[u(q_n(V), t^i_n(V), (V_i)|V_i(\omega) = v_i] \geq 0 \quad \text{for every } i \in \{1, \ldots, n\} \quad \text{and} \quad v_i \in \mathscr{V}.$$

**Corollary.** *Suppose that $\{G_n\}$ is an increasing sequence of groups of individuals that bargain over proposed changes through a sequence of feasible, incentive compatible, anonymous, consistent and admissible mechanisms as described in the theorem above. Suppose, in addition, that bargaining is voluntary; each mechanism $\langle q_n, t_n \rangle$ is individually rational. Then, implementing change becomes asymptotically impossible, or $Q_n \to 0$ in probability.*

The proof of the corollary is relegated to the next section. The intuition for it is as follows. Because the compensatory monetary transfers converge to constants they are necessarily arbitrary, and cannot be targeted to those who deserve them. Because the system of compensatory monetary transfers is constrained to be feasible, compensation cannot be given to all the individuals in the group. In a large group, some individuals who will suffer from the proposed change are sure to exist. A voluntary bargaining procedure respects the right of those individuals to block the proposed change.

## 3. PROOFS

The basic idea of the proof is that because of anonymity, the outcome of the bargaining process can only depend on aggregate indices of individuals' types. Our assumption about the structure of correlation implies that individuals' types are almost independent of any global indices. Therefore, as the group becomes larger, the outcome of the bargaining process cannot be highly correlated with the types of too many individuals, and must

12. This interpretation of the notion of voluntary bargaining ignores the possibility that individuals may object to an agreement that increases their utility in the hope of increasing it even more. This interpretation is standard in mechanism design literature. It abstracts from possibly many interesting problems, but it has the advantage of allowing us to focus on bargaining problems that arise under asymmetric information in contrast to "pure" bargaining or hold-up problems.

converge to a constant. Similarly, individuals' payments converge to deterministic functions of individuals' reports. Incentive compatibility then implies that the individuals' payments must converge to constants as well.

To prove the theorem, we state and prove three lemmas and a proposition. The first two lemmas record facts that are used in the proof of the third lemma. The third lemma forms the heart of the proof of the theorem. Its significance is explained below. It allows us to prove the proposition that establishes the convergence to constants. The proof of the theorem then follows as an easy corollary of incentive compatibility and feasibility.

In the following discussion, set inclusion should be interpreted as "$P$ a.s." The following lemma is a straightforward result of covering.

**Lemma 1.** *Let $\varphi$ be a covering transformation over $(\Omega, \mathcal{F}, P)$. For all nonnull events $B, C \in \mathcal{F}$, there exists a number $k \in \mathbb{N}$ such that $P(B \cap \varphi^k C) > 0$. If, in addition, $B \cap C = \varnothing$ and $B \cup C = \Omega$, then $P(B \cap \varphi C) > 0$.*

*Proof.* Let $F = \bigcup_{k \geq 1} \varphi^k C$. $P(C) > 0$ and $\varphi$ is covering, therefore, $P(F) = 1$. Suppose that $P(B \cap \varphi^k C) = 0$ for all $k \in \mathbb{N}$. $P(F) = 1$ implies $0 < P(B) = P(B \cap F) \leq \sum_{k=1}^{\infty} P(B \cap \varphi^k C) = 0$. A contradiction. Suppose now that $B \cap C = \varnothing$ and $B \cup C = \Omega$, but $B \cap \varphi C = \varnothing$. It follows that $\varphi C \subseteq C$. By iteratively applying $\varphi$ to both sides of $\varphi C \subseteq C$ we obtain $\varphi^n C \subseteq \cdots \subseteq C$ for every $n \in \mathbb{N}$. Hence, $P(\bigcup_{n \geq 1} \varphi^n C) = P(C) < 1$ a contradiction to covering. ||

Covering also implies,

*Definition.* A transformation $\varphi$ on $(\Omega, F, P)$ is incompressible if for every event $E \in F$, $\varphi^{-1} E \subseteq E$ implies $P(E \backslash \varphi^{-1} E) = 0$.

**Lemma 2.** *If a transformation $\varphi$ on $(\Omega, \mathcal{F}, P)$ is covering then it is incompressible.*

*Proof.* If $\varphi$ is not incompressible then there exists an event $E$ such that $\varphi^{-1} E \subseteq E$ and $P(E \backslash \varphi^{-1} E) > 0$. By iteratively applying $\varphi$ to both sides of $\varphi^{-1} E \subseteq E$ we obtain $\varphi^{-n} E \subseteq \cdots \subseteq E$ for every positive $n$. By iteratively applying $\varphi$ to both sides of $E = (E \backslash \varphi^{-1} E) \cup \varphi^{-1} E$ we obtain $E \subseteq \cdots \subseteq \varphi^m E$ for every positive $m$ and hence $\varphi^m (E \backslash \varphi^{-1} E) \subseteq \Omega \backslash \varphi^{m-1} E$ is disjoint from $E$ for every positive $m$. Hence $\varphi^m (E \backslash \varphi^{-1} E)$ and $\varphi^{-n} E$ are disjoint for every positive $n$ and $m$. Because $\varphi$ is covering there exists a positive $n$ such that $P(\varphi^{-n} E) > 0$. It therefore follows that $\bigcup_{n \geq 1} \varphi^m (E \backslash \varphi^{-1} E)$ does not cover the positive probability event $\varphi^{-n} E$, hence, $\varphi$ is not covering. A contradiction. ||

As mentioned above, the next lemma is the "heart" of the proof of the theorem. It is a generalization of Poincaré's Recurrence Theorem. Define the notion of recurrence as follows.

*Definition.* Fix a transformation $\varphi$ and an event $B \in F$. $\omega \in B$ is recurrent with respect to $\varphi$ and $B$ if there exists an integer $k \geq 1$ such that $\varphi^k \omega \in B$.

**Lemma 3.** *For any covering transformation $\varphi$ and event $B \in \mathcal{F}$, a.s. every $\omega \in B$ is infinitely recurrent with respect to $\varphi$ and $B$.*

*Proof.* The proof is adapted from Petersen (1983, pp. 33 and 39). Fix a covering transformation $\varphi$ and an event $B\in\mathcal{F}$. Let $B^*=\bigcup_{k=1}^{\infty}\varphi^{-k}B$. $\varphi$ is covering, therefore $\Omega\backslash\varphi^{-k}B=\varphi^{-k}(\Omega\backslash B)$ for every $k\in\mathbb{N}$ and

$$\begin{aligned}B\backslash B^* &= B\backslash\bigcup_{k=1}^{\infty}\varphi^{-k}B\\ &= B\cap[\Omega\backslash\bigcup_{k=1}^{\infty}\varphi^{-k}B]\\ &= B\cap[\bigcap_{k=1}^{\infty}(\Omega\backslash\varphi^{-k}B)]\\ &= B\cap\varphi^{-1}(\Omega\backslash B)\cap\varphi^{-2}(\Omega\backslash B)\cap\cdots,\end{aligned}$$

is the set of all the points of $B$ that are non-recurrent with respect to $\varphi$ and $B$. Note that if $\omega\in B\backslash B^*$ then $\varphi^n\omega\notin B\backslash B^*$ for all $n\geq 1$. Thus $(B\backslash B^*)\cap\varphi^{-n}(B\backslash B^*)=\varnothing$ for all $n\geq 1$ and hence $\varphi^{-k}(B\backslash B^*)\cap\varphi^{-(n+k)}(B\backslash B^*)=\varnothing$ for all $n\geq 1$ and $k\geq 0$. As a consequence, the sets $B\backslash B^*$, $\varphi^{-1}(B\backslash B^*)$, $\varphi^{-2}(B\backslash B^*),\ldots$ are pairwise disjoint.

Assume that $E=B\cup B^*$. By Lemma 2, $\varphi$ is incompressible and therefore $\varphi^{-1}E=\varphi^{-1}B\cup\varphi^{-1}B^*=B^*\subseteq E$ implies $P(E\backslash\varphi^{-1}E)=0$. But, $E\backslash\varphi^{-1}E=(B\cup B^*)\backslash B^*=B\backslash B^*$, so $P(B\backslash B^*)=0$. It follows that $\omega\in B$ is a.s. recurrent with respect to $\varphi$ and $B$.

We prove that $\omega\in B$ is a.s. infinitely recurrent with respect to $\varphi$ and $B$. Let $G$ be the set of all the points of $B$ which are not infinitely recurrent with respect to $B$.

$$G=B\cap\bigcup_{n=0}^{\infty}\varphi^{-n}(\Omega\backslash B^*).$$

We show that $P(G)=0$.

$$\begin{aligned}G &= B\cap\bigcup_{n=0}^{\infty}\varphi^{-n}(\Omega\backslash B^*)\\ &= B\cap[(\Omega\backslash B^*)\cup(\Omega\backslash\varphi^{-1}B^*)\cup(\Omega\backslash\varphi^{-2}B^*)\cup\cdots]\\ &= B\cap[(\Omega\backslash B^*)\cup(B^*\backslash\varphi^{-1}B^*)\cup(\varphi^{-1}B^*\backslash\varphi^{-2}B^*)\cup\cdots]\\ &= (B\backslash B^*)\cup[B\cap\bigcup_{n=0}^{\infty}(\varphi^{-n}B^*\backslash\varphi^{-(n+1)}B^*)].\end{aligned}$$

Now, by Lemma 2 $\varphi$ is incompressible and also $\varphi^{-1}\varphi^{-n}B^*=\varphi^{-(n+1)}B^*\subseteq\varphi^{-n}B^*$, so $P(\varphi^{-n}B^*\backslash\varphi^{-(n+1)}B^*)=0$ for every $n=0,1,2,\ldots$ Since we already proved that $P(B\backslash B^*)=0$ it follows that $P(G)=0$. ||

The importance of Lemma 3 is in that it implies that it is impossible to partition $\Omega$ into two disjoint events where $V_1,\ldots,V_n$ have different distributions of realizations for infinitely many $n$s. Anonymity then implies that is impossible to divide $\Omega$ into two disjoint events where $Q$ obtains different values. This is proved in the next proposition.

Before we state the proposition, note that consistency implies that the sequences of random variables $\{Q_n\}$ and $\{T_n^i\}$, $i\in\mathbb{N}$, are fundamental in measure and therefore converge in probability and contain subsequences that converge with probability 1 (see, *e.g.* Halmos (1950, p. 93)). Denote their limits by $Q$ and $T^i$, respectively. We have.

**Proposition.** *$Q$ is a constant a.s. and for every $i\in\mathbb{N}$, the random variable $T^i$ depends only on the realization of $V_i(\omega)$. That is, $i$'s report deterministically determines the size of her transfer.*

*Proof.* Suppose that $Q$ is not a constant a.s. It follows that there exists an event $A\in\mathcal{F}$, with $0<P(A)<1$, such that $Q$ obtains different values on $A$ and on $A$'s complement.

*Definition.* $\omega$ and $\omega'$ are $n$-equivalent if for every $v \in \mathscr{V}$, the number of appearances of $v$ among $V_1(\omega), \ldots, V_n(\omega)$ is equal to the number of its appearances among $V_1(\omega'), \ldots, V_n(\omega')$.

Note that for a.s. all $\omega \in A$ and $\omega' \in \Omega \backslash A$, $\omega$ and $\omega'$ cannot be $n$-equivalent for an infinite number of $n$'s, because whenever $\omega$ and $\omega'$ are $n$-equivalent, anonymity implies that $Q_n(\omega) = Q_n(\omega')$ and if $\omega$ and $\omega'$ are $n$-equivalent for an infinite number of $n$'s, $Q(\omega) = \lim_{n \to \infty} Q_n(\omega) = \lim_{n \to \infty} Q_n(\omega') = Q(\omega')$ in contradiction to $Q$ obtaining different values on $A$ and on $\Omega \backslash A$.[13]

Thus, $P(\varphi A \cap (\Omega \backslash A)) > 0$. Let $B = \varphi A \cap (\Omega \backslash A)$. Note that $\varphi^{-1} B \subseteq A$. Pick $\omega \in \varphi^{-1} B$ and let $\omega' = \varphi\omega \in \Omega \backslash A$. By assumption for a.s. all $\omega \in \varphi^{-1} B$, $\omega$ and $\omega' = \varphi\omega$ cannot be $n$-equivalent for an infinite number of $n$'s. We show that they must be and obtain a contradiction.

For $\omega$, the profile of individuals' types $V_1, \ldots, V_n$ is given by

$$\omega: (V_1(w), \ldots, V_n(w)).$$

For $\omega' = \varphi\omega$, the profile of individuals' types is given by

$$\begin{aligned} \omega': &(V_1(w'), \ldots, V_n(w')) \\ &= (V_1(\varphi\omega), \ldots, V_n(\varphi\omega)) \\ &= (V_2(\omega), \ldots, V_{n+1}(\omega)), \end{aligned}$$

because by definition, $V_k(\omega) = V_1(\varphi^k \omega)$ for all $k \in \mathbb{N}$ and $\omega \in \Omega$. Therefore, in order to show that $\omega$ and $\omega'$ are $n$-equivalent for an infinite number of $n$'s we must show that $V_1(\omega) = V_{n+1}(\omega) = V_1(\varphi^n \omega)$ for an infinite number of $n$'s. Suppose that $V_1(\omega) = v_1$ and let

$$B_{v_1} = \{\omega \in A \colon V_1(\omega) = v_1\}.$$

The proof follows from the fact that by Lemma 3, a.s. every $\omega \in B_{v_1}$ is infinitely recurrent with respect to $\varphi$ and $B_{v_1}$.

We now prove the second part of the proposition. The proof is similar to the proof employed to prove the first part of the proposition. Fix an $i \in \mathbb{N}$ and suppose that $T^i$ is not deterministically determined by $V_i$. For every $v_i \in \mathscr{V}$, let $C(v_i) = \{\omega \colon V_i(\omega) = v_i\}$ denote the event where $i$'s valuation is $v_i$. As before, if $T^i$ is not determined by $V_i$ then there must exist a type $v_i \in \mathscr{V}$ and an event $A \subseteq C(v_i)$ such that $0 < P(A) < P(C(v_i))$ and such that $T^i$ obtains different values on $A$ and on $C(v_i) \backslash A$.

*Definition.* Given $i \in \mathbb{N}$ and $v_i \in \mathscr{V}$, $\omega$ and $\omega'$ are $n$-equivalent with respect to $i$ and $v_i$ if $V_i(\omega) = V_i(\omega') = v_i$ and for every $v \in \mathscr{V}$, the number of appearances of $v$ among $V_1(\omega), \ldots, V_n(\omega)$ is equal to its number of appearances among $V_1(\omega'), \ldots, V_n(\omega')$.

As before, for a.s. all $\omega \in A$ and $\omega' \in C(v_i) \backslash A$, $\omega$ and $\omega'$ cannot be $n$-equivalent with respect to $i$ and $v_i$ for an infinite number of $n$'s. By Lemma 1, there exists an integer $k \geq 1$ such that $P(\varphi^k A \cap (C(v_i) \backslash A)) > 0$. Let $B = \varphi^k A \cap (C(v_i) \backslash A)$. As before, $\varphi^{-k} B \subseteq A$. Pick an $\omega \in \varphi^{-k} B$ and let $\omega' = \varphi^k \omega \in C(v_i) \backslash A$. By assumption, for a.s. all $\omega \in \varphi^{-k} B$, $\omega$ and $\omega'$ cannot be $n$-equivalent for an infinite number of $n$'s. We show that they must be and obtain a contradiction.

13. This is the only place where consistency is used in the proof.

As before, for $\omega$, the profile of individuals' types $V_1, \ldots, V_n$ is given by

$$\omega : (V_1(w), \ldots, V_n(w)).$$

For $\omega' = \varphi^k \omega$, the profile of individuals' types is given by

$$\begin{aligned}\omega' : (V_1(w'), \ldots, V_n(w'))\\ = (V_1(\varphi^k\omega), \ldots, V_n(\varphi^k\omega))\\ = (V_{1+k}(\omega), \ldots, V_{n+k}(\omega)).\end{aligned}$$

Therefore, in order to show that $\omega$ and $\omega'$ are $n$-equivalent for an infinite number of $n$'s, it is enough to show that $(V_1(\omega), \ldots, V_k(\omega)) = (V_{n+1}(\omega), \ldots, V_{n+k}(\omega)) = (V_1(\varphi^n\omega), \ldots, V_1(\varphi^{n+k-1}\omega))$ for an infinite number of $n$'s. Suppose that $(V_1(\omega), \ldots, V_k(\omega)) = (v_1, \ldots, v_k)$ and let

$$B_{v_1,\ldots,v_k} = \{\omega \in A : (V_1(\omega), \ldots, V_k(\omega)) = (v_1, \ldots, v_k)\}.$$

The proof follows from the fact that by Lemma 3, $\omega \in B_{v_1,\ldots,v_k}$ is a.s. infinitely recurrent with respect to $\varphi$ and $B_{v_1,\ldots,v_k}$. ||

We are now in a position to prove our main result.

*Proof of Theorem.* By the proposition, $Q$ equals a constant a.s., denote it $q$. Therefore if individual $i$ reports $\hat{v}_i \in \mathscr{V}$ instead of her real valuation $V_i(\omega) = v_i$ it does not affect the mechanism's decision. By the proposition, for every $i \in \mathbb{N}$, the value of $T^i$ depends only on the realization of $V_i$, denote it $t^i(v_i)$. Hence, individual $i$'s monetary transfer depends only on her report to the mechanism $\hat{v}_i \in \mathscr{V}$, not on her real valuation $v_i$, and not on other individuals' valuations or reports to the mechanism.[14]

Incentive compatibility implies

$$E[u(Q_n, T^i_n, V_i) \mid V_i(\omega) = v_i] \geqq E[u(q_n(\hat{v}_i, V_{-i}(\omega)), t^i_n(\hat{v}_i, V_{-i}(\omega)), V_i) \mid V_i(\omega) = v_i]$$

for all $i \in \{1, \ldots, n\}$, $v_i, \hat{v}_i \in \mathscr{V}$, and $n \in \mathbb{N}$. The fact that $\{Q_n\}$ converges to a constant and $\{T^i_n\}$ converges to $t^i(V_i(\omega))$ implies that $\lim_{n\to\infty} q_n(\hat{v}_i, V_{-i}(\omega)) = q$ a.s. and also $\lim_{n\to\infty} t^i_n(\hat{v}_i, V_{-i}(\omega)) = t^i(\hat{v}_i)$ a.s. for all $\hat{v}_i \in \mathscr{V}$. Continuity of $u$ implies that $\lim_{n\to\infty} u(q_n(\hat{v}_i, V_{-i}(\omega)), t^i_n(\hat{v}_i, V_{-i}(\omega)), v_i) = u(q, t^i(\hat{v}_i), v_i)$ a.s. for all $i \in \mathbb{N}$ and $v_i, \hat{v}_i \in \mathscr{V}$. Applying the dominated convergence theorem to the previous inequality yields

$$u(q, t^i(v_i), v_i) \geqq u(q, t^i(\hat{v}_i), v_i).$$

Now, because $u$ is increasing in its second argument, it follows that $t^i(v_i) \geqq t^i(\hat{v}_i)$ for any pair $v_i, \hat{v}_i \in \mathscr{V}$, from which it follows that $t^i(v_i)$ is independent of $v_i$. This completes the proof of the theorem. ||

*Proof of Corollary.* By the previous theorem $\{Q_n\}$ converges to a constant $q$ and $\{T^i_n\}$ converges to a constant $t^i$ for all $i \in \mathbb{N}$. Apply the dominated convergence theorem to the individual rationality constraints to get in the limit

$$u(q, t^i, v_i) \geqq 0 \qquad \text{for all } v_i \in \mathscr{V}.$$

14. We implicitly assume here that the mechanism punishes individuals if they report an impossible vector of valuations sufficiently harshly to deter such behaviour from occurring.

Suppose that $q > 0$. The fact that for some $v_i \in \mathcal{V}$, $u(q, 0, v_i) < 0$, implies that it must be the case that $t^i > 0$ for all $i \in \mathbb{N}$, a contradiction to feasibility. Therefore, it follows that $q = 0$ and $t^i = 0$ for all $i \in \mathbb{N}$. ||

## 4. DISCUSSION

### 4.1. *Is the assumption of covering necessary?*

We show that the theorem above cannot be extended to the more general case where individuals' types are mixtures of ergodic random variables. The simplest such case is where individuals' information is exchangeable as in Example 1, case (i).

*Example* 7. Suppose that individuals' utilities are given by the function $u(q, t^i, v_i) = q \cdot v_i + t^i$ and that individuals' types are either all 1 with probability $p \in (0, 1)$ or $-1$ with probability $1 - p$. Consider the following sequence of mechanisms.[15]

$$q_n(v_1, \ldots, v_n) = \begin{cases} 1, & \text{if } v_1 = v_2 = \cdots = v_n = 1, \\ 0, & \text{otherwise,} \end{cases}$$

$$t^i_n(v_1, \ldots, v_n) = \begin{cases} 0, & \text{if } v_1 = v_2 = \cdots = v_n = 1, \\ -1, & \text{otherwise.} \end{cases}$$

It is straightforward to verify that these mechanisms are incentive compatible, individually rational, anonymous, consistent and admissible, yet the degree of reform does not converge to 0. The intuition for why the theorem fails when individuals' information is exchangeable is that the joint distributions of any two individuals $i$ and $j$ is independent of their identity. Any individual $i$ has the same belief about any other individual $j$. Consequently, the central planner can use the report of just any individual $j$ to construct an anonymous lottery that will induce $i$ to reveal her type without giving her any "informational rents" as in Crémer and McLean (1985, 1988) and McAfee and Reny (1992).

The fact that radically different results obtain under exchangeable and ergodic information structures raises the question of which is more plausible. Under the former, any individual $i$ has exactly the same belief about any other individual $j$ and has a type that is significantly correlated with aggregate group parameters. On the other hand, when individuals' types are ergodic, individuals are likely to hold different beliefs about different individuals and have a type that is almost independent of aggregate parameters of the group. We believe that these two characteristics make ergodic information structures more plausible. Casual empiricism confirms that, in fact, individual do hold different beliefs about different individuals. And, as explained above in Section 2, the spread of mass media allows everyone equal access to credible public information about the group's characteristics and as a consequence their opinions or beliefs about aggregate parameters should not depend on their types. The difference boils down to whether a *single* individual's type conveys any information about aggregate parameters of the group. This perhaps may be the case in a totalitarian regime where communication channels are tightly controlled and individuals are all likely to hold similar opinions. But the proliferation and fragmentation of the media in a democracy implies that this could hardly be the case in a freer society.

15. These mechanisms are called "shoot them all" mechanisms by Fudenberg and Tirole (1991).

### 4.2. *Is anonymity necessary and sufficient?*

The assumption of anonymity is obviously necessary for our results. Consistency, by itself, is not sufficient to ensure convergence to a constant. Without anonymity, the decision can depend only on a small sample of the population, for example, it may give individual 1 dictatorial powers. In such a case, her decision will obviously not be a constant.

As for sufficiency, intuitively, the assumption of anonymity implies that adding one more individual to an already large group should not affect the outcome by much. This suggests that anonymity, by itself, should be sufficient to guarantee convergence of the sequence of outcomes thus obviating the need for the consistency assumption. An example due to Blackwell and Freedman (1964) shows that this is not generally the case. (However, see the Appendix for an example where anonymity alone is sufficient to guarantee convergence.)

### 4.3. *Related literature*

Our work is closely related to the work of Rob (1989) and especially Mailath and Postlewaite (1990) that showed, for a subclass of the bargaining problems studied here that under independent asymmetric information, the probability of providing a public good tends to zero as the number of individuals increases, although efficiency might require providing the good with probability one. The two main differences between this work and theirs is that we allow for more general utility functions and for the existence of correlation among individuals' preferences and information. Mailath and Postlewaite (1990) prove a theorem similar to the one presented here for the case where individuals' types are independent and their utilities are given by $u(q, t, v) = q \cdot v + t$ without assuming anonymity, consistency, or admissibility, and provide a bound on the rate of convergence as well. When individuals' types are independent, anonymity is without loss of generality. In this case, our result generalizes Mailath and Postlewaite (1990) in that it allows for general utilities but is more special in that it requires consistency and admissibility, or alternatively, that some restrictions be imposed on the families of mechanisms that are being considered (see Footnote 9). More generally, Mailath and Postlewaite (1990) restrict the type of environment but allow for general mechanisms whereas we allow for a more general environment but restrict the class of mechanisms. Another recent contribution that is concerned with similar problems to those discussed here is Al-Najjar and Smorodinsky (1997) who define a general measure of agents' influence and use it, among other things, to demonstrate the impossibility of providing public goods. They also consider a model where individuals' types are correlated, therefore, in order to eliminate the type of lotteries used by Crémer and McLean (1985, 1988), they focus on bargaining processes that satisfy the more demanding notion of *ex post* individual rationality.

Another paper that presents a theory of resistance to change that is based on asymmetric information is Fernandez and Rodrik (1991). Their approach differs from ours in two important respects. First, they do not allow for the possibility of arranging compensatory monetary transfers and second, they focus on majority rule, a bargaining procedure that is not necessarily voluntary, as the mechanism to implement change. The reason they provide for the difficulty of implementing change is different from ours. They show that although a proposed change may enhance welfare and win the support of the majority of the population under complete information, uncertainty about the indentity of those who will gain from the proposed change may be such that the majority of the population will vote against the proposed change.

A vast less formal literature investigates the sources of opposition to change. Kuran (1988) presents a survey of theories of collective conservatism.

## 5. CONCLUSION

Our results lead to two conclusions that pertain to political economy. First, the difficulty of aggregating information in large groups highlights the need to rely on smaller groups to efficiently aggregate information and implement decisions. In a recent paper, Spector (1996) suggested this as a possible explanation for the development and success of the notion of representative democracy. Second, our results serve to explain some of the difficulties involved with implementing reforms, economic or others. Standard explanations emphasize balance of power theories. By virtue of their incumbency, the defenders of the old order or the status quo are well organized and politically powerful, while the supporters of change are dispersed and politically weak. The latter cannot overcome the opposition mounted by the former and as a consequence, the reform attempts fail. Such explanations certainly capture some of the difficulties associated with implementing changes, yet they are subject to the following simple economic criticism: if a proposed change is Pareto efficient, the gains from change outweigh the losses. It follows that "gainers" should be able to compensate losers in such a way that everybody benefits and the proposed change enjoys unanimous support.[16] Why, then, do efficient changes fail or take so long to be implemented? We hope that our results provide some insight into the answer to this question.

## APPENDIX: AN EXAMPLE

The following example illustrates our main result for a specific simple ergodic environment. Restricting our attention to this simple environment allows us to obtain sharper results (including a bound on the rate of convergence) and permits us to dispense with the consistency assumption. The method of proof is different from the one used in the proof of the main result. It follows the treatment in Mailath and Postlewaite (1990, Appendix 2) and it relies on a Central Limit Theorem for dependent variables. It suggests that imposing bounds on the rate at which the correlation between individuals' types decreases to zero would allow us to obtain similar results in more general environments as well.

Suppose that individuals' utilities are given by the function $u(q, t^i, v_i) = q \cdot v_i + t^i$. Let $X_1, X_2, \ldots$ be a sequence of i.i.d. r.v.s. where, for every $i$, $P(X_i = -1) = P(X_i = 1) = \frac{1}{2}$. For every $i \in \mathbb{N}$, let $V_i = \max\{X_i, X_{i+1}\}$. Thus, $V_i$ is "high" with probability $\frac{3}{4}$ and "low" with probability $\frac{1}{4}$. Every individual's type is correlated with the types of his immediate neighbours ($\operatorname{cov}(Y_i, Y_{i+1}) = 0{\cdot}29\ \forall i \in \mathbb{N}$) but is independent of the types of all other individuals ($\operatorname{cov}(Y_i, Y_{i+k}) = 0$ for $k \geqq 2$). Thus, as explained in Example 4 above, the sequence $V_1, V_2, \ldots$ is stationary and ergodic. Furthermore, since $E[V_i] = \frac{1}{2} > 0$, as the group becomes larger, the probability that implementing change is efficient increases to 1.

We retain the assumptions of feasibility, anonymity, and individual rationality. We dispense with consistency, but strengthen admissibility to require that individuals cannot be forced to pay more than some possibly large but finite amount $T < \infty$.

Consider a feasible and anonymous mechanism $\langle q_n, t_n \rangle$. Anonymity implies that $q_n$ may depend only on $k \in \{0, \ldots, n\}$ – the number of individuals who reported a low type; $t_n^i$ may depend on $i$'s report and on $k$. Without loss of generality, we may assume that the mechanism prescribes the same payment function to all individuals.[17] Feasibility and individual rationality imply that this payment function must be given by

$$t_n^i(\hat{v}) = \begin{cases} q_n(k) + x_n(k), & \hat{v}_i = -1,\ 1 \leqq k \leqq n, \\ -q_n(k)(\alpha n/n-k) + y_n(k), & \hat{v}_i = 1,\ 0 \leqq k \leqq n, \end{cases}$$

16. See, *e.g.* Rodrik (1996).

17. If there exists a non-symmetric mechanism that succeeds in implementing change, then so must the symmetric mechanism that is the average of the non-symmetric mechanism over permutations of individuals' identities.

where $\alpha n, \alpha \in (0, 1)$, is the expected total payment made by individuals who report high values when change takes place, and $\sum_{k=1}^{n} P(k \mid V_i = -1)x_n(k) = \sum_{k=1}^{n} P(k \mid V_i = 1)y_n(k) = 0$.[18]

Incentive compatibility for an individual with a high type (for whom the incentive constraint is also more likely to be binding is

$$\sum_{k=0}^{n} P(k \mid V_i = 1)\left(q_n(k)\left(1 - \frac{\alpha n}{n-k}\right) + y_n(k)\right) \geq \sum_{k=0}^{n-1} P(k \mid V_i = 1)(2q_n(k+1) + x_n(k+1)).$$

This implies

$$\sum_{k=1}^{n+1} (P(k \mid V_i = -1) - P(k-1 \mid V_i = 1))x_n(k) + 2\sum_{k=0}^{n} P(k \mid V_i = 1)(q_n(k) - q_n(k+1))$$
$$\geq \sum_{k=0}^{n} P(k \mid V_i = 1)q_n(k)\left(\frac{\alpha n}{n-k}\right), \qquad (**)$$

where $P(n+1 \mid V_i = -1) = q_n(n+1) = x_n(n+1) = 0$.

A Central Limit Theorem for dependent variables (Durrett, 1991, Thm. 7.6, pp. 375–376) implies that for large values of $n$, $k$ is approximately normally distributed with a variance of $cn$ where $c$ is some positive, and finite, constant. (The theorem requires that the correlations between individuals' types decrease to zero at a fast enough rate—a condition that is satisfied in this example.) Denote the most probable realization of $k$ by $k^*$. Thus, for large $n$, $k^*$ is such that $P(k-1) \leq P(k)$ if $k \leq k^*$ and $P(k-1) > P(k)$ if $k > k^*$. Note also that since individuals of high, low, or unspecified types hold the same beliefs on all but at most two individuals, for every integer $k$, both $P(k \mid V_i = -1)$, $P(k \mid V_i = 1) \to_{n \to \infty} P(k)$. Thus, for large enough $n$ also $P(k-1 \mid V_i = 1) \leq P(k \mid V_i = -1)$ if $k \leq k^*$ and $P(k-1 \mid V_i = 1) > P(k \mid V_i = -1)$ if $k > k^*$. Consequently,

$$\sum_{k=1}^{n+1} (P(k \mid V_i = -1) - P(k-1 \mid V_i = 1))x_n(k)$$
$$\leq T[\,|P(k \leq k^* \mid V_i = -1) - P(k \leq k^* \mid V_i = 1)| + |P(k > k^* \mid V_i = 1) - P(k > k^* \mid V_i = -1)|\,], \qquad (***)$$

and since individuals of high, low, or unspecified types hold the same beliefs on all but at most two individuals, the last expression is smaller or equal to $4TP(k^*)$.

Straightforward arguments given in Mailath and Postlewaite (1990, Appendix 2) imply that

$$\sum_{k=0}^{n} P(k \mid V_i = 1)(q_n(k) - q_n(k+1)) \leq P(k^*).$$

Therefore, the RHS of (**) is smaller or equal to $(4T+1)P(k^*)$, and larger or equal to $\alpha \sum_{k=0}^{n} P(k \mid V_i = 1)q_n(k)$.

Thus, the *ex ante* probability that change is implemented, $\sum_{k=0}^{n} P(k)q_n(k)$, which by an argument similar to the one given in (***) above is close to $\sum_{k=0}^{n} P(k \mid V_i = 1)q_n(k)$ when $n$ is large, is bounded from above by $((4T+1)/\alpha)P(k^*)$. Finally, the fact that for large $n$, $k$ is approximately normally distributed with variance $cn$ implies that $P(k^*)$ decreases to zero at a rate proportional to $1/\sqrt{n}$.

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18. Requiring instead that $\sum_{k=1}^{n} P(k \mid V_i = -1)x_n(k) \geq 0$ and $\sum_{k=1}^{n} P(k \mid V_i = -1)y_n(k) \leq 0$ would not affect the results. Notice also that the individual rationality constraint for the high type may impose further restriction on $\alpha$ and $y_n(\cdot)$.

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